Access Unit 1: Place value, ordering and rounding 1

Objectives

a) **Count, read, write and order whole numbers to 100 and know what each digit represents (including 0 as a place holder).**

b) **Place numbers on a number line.**
Start with all the numbers having a marker, gradually reduce the markers so that student has to begin to estimate, e.g. where 36 will be placed between 30 and 40.

c) **Round two-digit numbers to the nearest 10.**
Support with a number line with markers for 5s.

d) **Describe and extend simple number sequences (including odd/even numbers, counting on or back in ones or tens from any two-digit number).**
Support with a 0-100 square and use a calculator to show the effect of adding 10 to any number, and that counting back is equivalent to subtracting 1.

e) **Use symbols: less than (<), greater than (>), equals (=).**
Stress that '=' means 'equals', not just 'the answer is'.

Keywords

units, ones, tens, digit, one-digit number, two-digit number, order, place value, zero, nought, odd, even, round, number line, count on, count back, greater than, less than, equals

Common misconceptions

Can read numbers up to 100, but has not understood the significance of the place value of each digit.
Use base 10 apparatus to make 'pictures' of numbers and ask them to select the tens and units for, say, 43.

Thinks 23 is even because of the '2'.

Only understands '=' as 'the answer is'.

Place value, ordering and rounding 1 check-up

1. Read these numbers and put them in order from smallest to largest:

23, 17, 8, 55, 82, 70

Round them to the nearest 10.

What is the value of 2 in 23?

In 82?

Which of these numbers are odd?

Even?

2. Count back in 1s from 17.

3. Count on in 10s from 23.

4. For each pair of numbers, put < or > as appropriate:

23 17

8 55

82 70

Place value, ordering and rounding 1

Opportunities for reasoning/problem solving – teacher notes

Thinking of a number

Students may need a 100 square to help with these.

Answers

1. 48

2. 25

3. 47

4. 88

5. 20

6. 78

7. 58

8. 37, 47, …, 97

100 square – shaded numbers

If a student does not know how to start with this problem, ask them questions about the 100 square such as, 'What happens to the numbers as you go across the 100 square?' and 'What happens as you go down?'

If they cannot answer, take away the puzzle sheet and look at a 100 square with them, asking the questions again.

Then take away the 100 square and give them the puzzle sheet. Suggest they start by going across from the given numbers – going up and down in 1s. If they still cannot do the vertical columns, ask again what happens to the numbers as you go up and down in a column.

100 square – missing numbers

This follows from the previous activity, but is more difficult because the pieces are not in the correct places. The same questions and support may be needed in order for the student to be able to make a start.

Thinking of a number

What are these numbers that I'm thinking of?

1. I'm thinking of a number between 40 and 50. It is even and would be rounded to 50. There are two numbers which it could be – it is the larger one.

2. This number is > 20 and < 30. It is in the 5 times table.

3. My next number is 1 less than the first number.

4. I'm thinking of a two-digit number where both the digits are the same. It is the largest even number it could be.

5. This number is in the 10 times table. It is > 10 and < 30.

6. This number is 10 less than the answer to question 4.

7. This number is 10 more than the answer to question 1.

8. If I start at 27 and keep adding 10, what numbers will I get which are < 100?

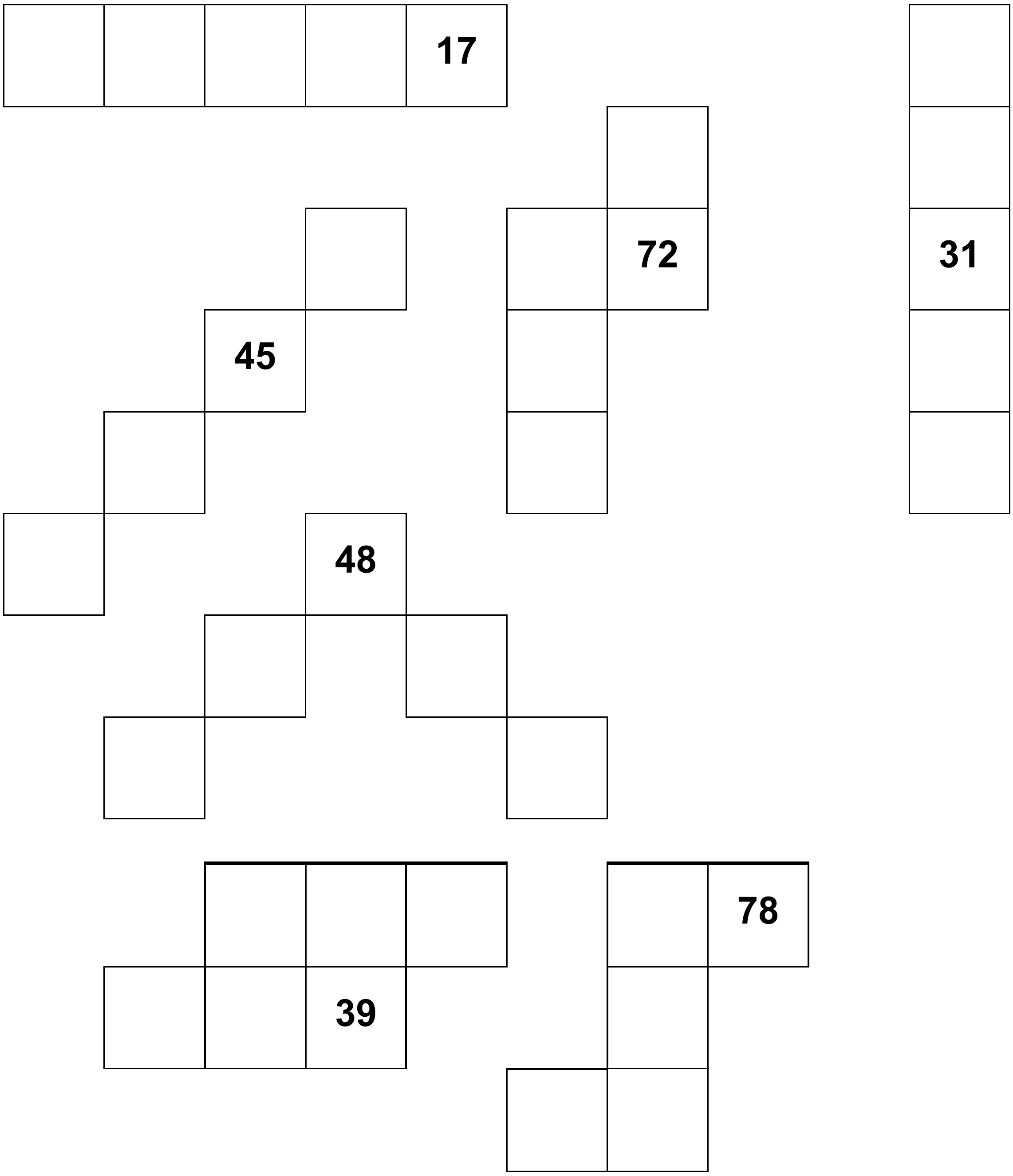
100 square – shaded numbers

This is a normal 100 square but most of the numbers are missing.
Can you work out which numbers go in the shaded squares?

						27			
			44						
								59	
					76				
	82								

100 square – missing numbers

These pieces have been cut out of a 100 square. Can you work out the numbers which go in the empty boxes?



Access Unit 2: Place value, ordering and rounding 2

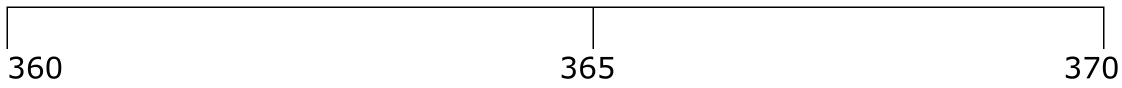
Objectives

a) **Read, write and order whole numbers to at least 1000 and know what each digit represents.**

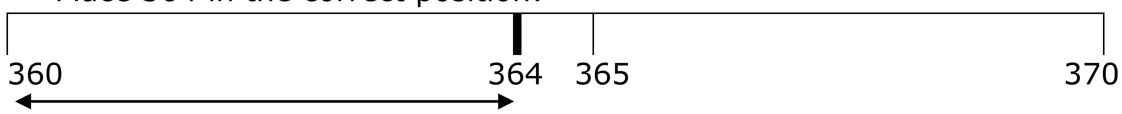
b) **Count on or back in tens or hundreds from any two- or three-digit number.**
Use a calculator to show what happens if this is difficult to understand.

c) **Use symbols: less than (<), greater than (>), equals (=).**

d) **Round any positive integer less than 1000 to the nearest 10 or 100. Support with number lines:**
e.g. Round 364 to the nearest 10.
364 is between 360 and 370, so draw these on a number line with 365 in the middle:

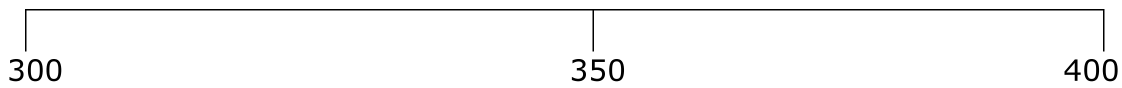


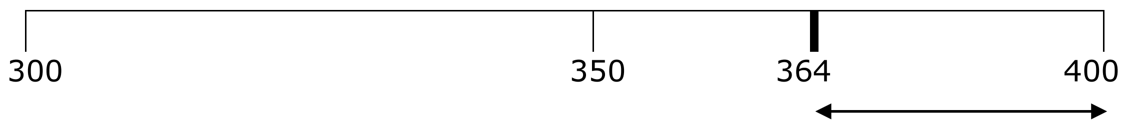
Place 364 in the correct position:



It can be seen that 364 is nearer to 360 than 370 and therefore is rounded to 360.

e.g. Round 364 to the nearest 100.
364 is between 300 and 400:





It can be seen that 364 is nearer to 400 and therefore it is rounded to 400.

Keywords
units, ones, tens, **hundreds, thousands**, digit, one-digit, two-digit number, order, place value, zero, nought, odd, even, round, number line, count on, count back, greater than, less than, equals

Common misconceptions

Writes ‘five hundred and six’ as 5006 or 56, not understanding ‘0’ as a place holder.

Always rounds up, uses the language ‘round up to’.

Place value, ordering and rounding 2 check-up

1. Given the digits 2, 5, 7, make all the possible three-digit numbers and put them in order. Say which are even/odd, round them to nearest 10 and 100, say what the 2 represents in each number. Take a pair of the numbers and write them with < or > as appropriate.

Number	Even/odd	Round to nearest 10	Round to nearest 100	What value does the 2 have?

2. Count back in 10s from 538.

3. Count on in 100s from 139.

4. Put 251 into the calculator. What do you have to key in to replace the 5 with 0? What about the 2?

Place value, ordering and rounding 2

Opportunities for reasoning/problem solving – teacher notes

Maximum number

Roll a dice. With each roll, the student writes the number in one of the boxes, aiming to make the biggest number possible. (This is will a combination of luck and strategy.) After reading the number obtained, they can rearrange the digits to make the largest number possible (if their original number is not already the largest possible), and read that.

They can have another go, if required.

Missing numbers in a pattern

The key to this puzzle is realising that the numbers go across in 1s and down in 10s. If the student finds this difficult, they may find a 100 square a useful aid.

Answers

223		**125**	**126**	127	**128**	129			
233								**485**	**486**
243		362					**494**	495	
253		**372**	**373**	**374**	**375**		**504**		
263									
		698		**575**			**848**	**849**	
706	**707**	708			586			859	**860**
716						**597**			**870**

Maximum number

Your teacher will roll a 6-sided dice and you can write each number in one of the boxes. You are trying to get the biggest number possible.

Once you have written a number in a box you cannot change it, so think carefully about the best place to put it.

Read the number you have made.

Is it the biggest possible number you could make with those four digits?

If not, rearrange the digits to make the largest possible number here:

Now read that number.

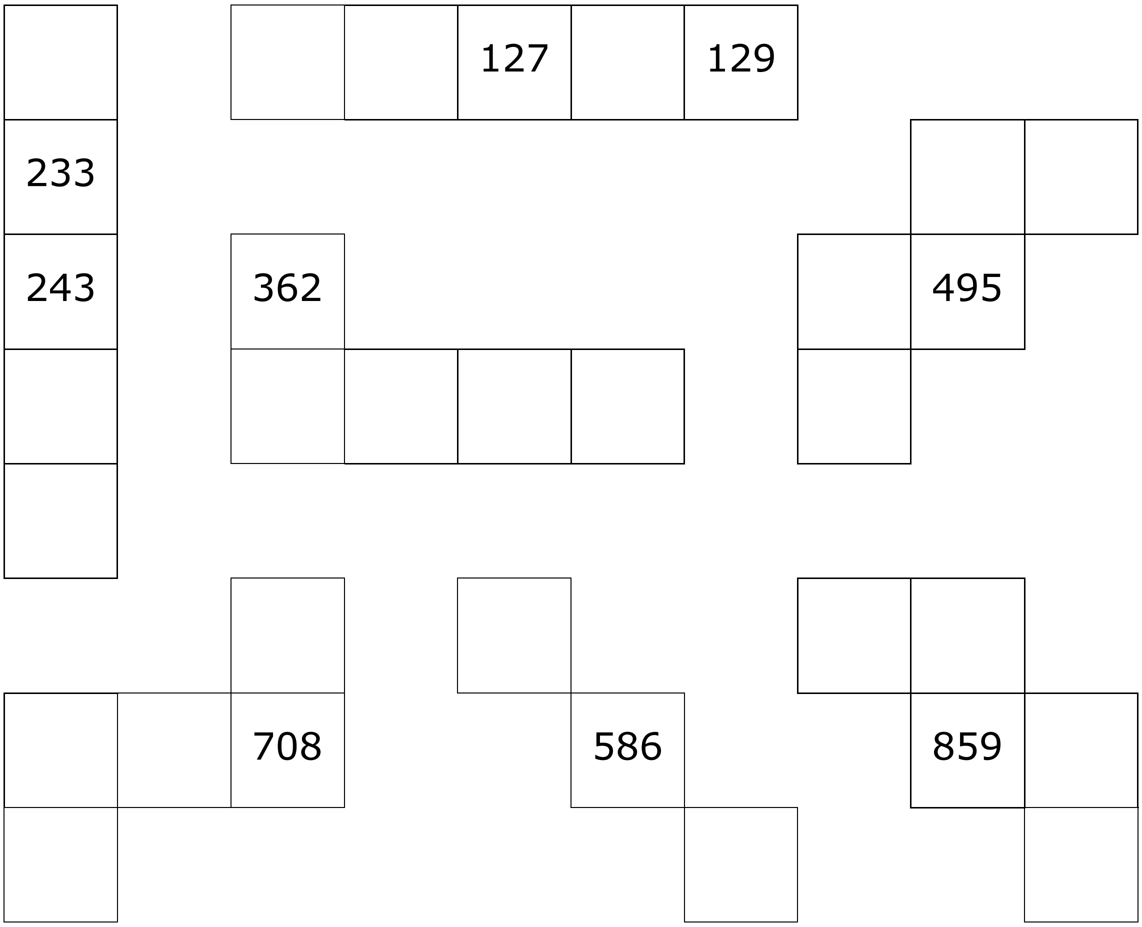
If you would like another go, do that here:

And rearrange it if necessary here:

Missing numbers in a pattern

All of these pieces are taken from a grid with the numbers from 1 to 1000.

Look carefully at the shapes with two numbers to discover the patterns, then fill in the empty boxes with the missing numbers.



Access Unit 3: Place value, ordering and rounding 3

Objectives

a) **Order a given set of positive and negative integers, including placing them on a number line.**
   Use horizontal and vertical number lines.

Keywords

units, ones, tens, hundreds, thousands, digit, one-digit number, two-digit number, order, place value, zero, nought, odd, even, round, number line, count on, count back, greater than, less than, equals, **negative, integer, horizontal, vertical, difference**

Common misconceptions

Has not understood that as negative numbers get smaller, the numerical element gets bigger.

Number lines can be very helpful here.
Vertical number lines can be related to a lift in a building (above and below ground level). There is also a useful connection to be made with co-ordinates in 4 quadrants.

Place value, ordering and rounding 3 check-up

1. Draw a number line from –10 to +10 and place these numbers on it:

–2, 0, +6, –7, +5, –8, +1, –4

2. If these numbers represent temperatures and on one winter's day it is 1°C in London and –4°C in New York, what is the temperature difference?

Place value, ordering and rounding 3

Opportunities for reasoning/problem solving – teacher notes

Making target numbers

These questions give the student practice in moving up and down the number line as they add and subtract.

Answers

1. 6
2. 8
3. –9
4. 0
5. 6°C
6. 9°C

Making target numbers

Draw a number line from −10 to +10 to help with these questions.

1. What do I need to add to −3 to get to +3?

2. What do I need to subtract from 7 to get to −1?

3. If I start at −8 and add 2, then subtract 3, where am I now?

4. I start at 4, subtract 6 and then add 2. Where am I now?

Draw a vertical number line to represent the temperatures from −5°C to +5°C to help with these questions.

5. In winter the temperature in Manchester rises to +4°C during the day, and falls to −2°C at night. How much has the temperature fallen?

6. A fridge keeps food at −5°C. I take some tomatoes out to let them warm up before I eat them. Half an hour later they are at +4°C. How many degrees warmer are they?

Access Unit 4: Addition and subtraction 1

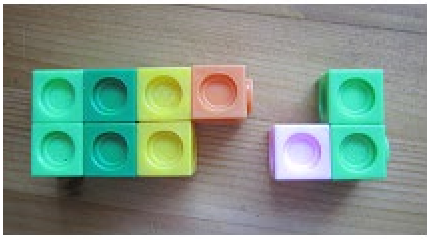
Objectives

a) Know by heart all addition and subtraction facts for each number to 10, then 20.

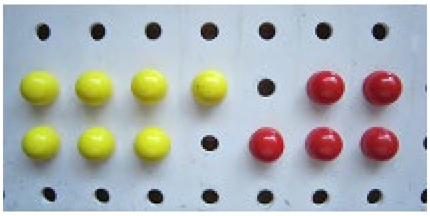
b) Understand the effect of adding and subtracting combinations of even and odd numbers.

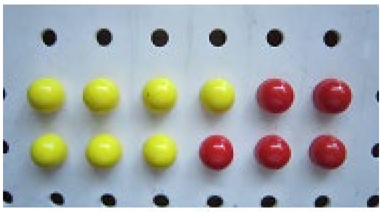
Ask the student to make two odd numbers using linking cubes, matching up the pairs, and leaving the odd ones to show that they are an odd numbers. Put them together (adding) to show what happens to the odd ones when the numbers are added together – it becomes an even number.
Ask what they think will happen with a different pair of odd numbers. If they are still not sure, repeat the practical example.

e.g. 7 and 3 added together make 10 – the pairs here are made clearer by the use of colour.



Alternatively, or additionally if more experience is needed, repeat with pegs on a pegboard.
e.g. 7 add 5 equals 12





Subtraction can be shown in a similar way with cubes by laying the smaller odd number on top of the larger odd number with the odd ones matched up. The result of the subtraction is the part not covered – an even number.

c) Understand that subtraction is the inverse of addition; state the subtraction corresponding to a given addition and vice versa.

Ask the student to make up an addition sentence with numbers which are less than 10, e.g. 5 + 6 = 11.

Ask them for another addition, using the same numbers. (6 + 5 = 11) If they can do this they have some understanding that addition can be done in any order.

Then ask for two subtractions using the same numbers. They may need prompting to start with 11. (11 – 5 = 6, 11 – 6 = 5)

For some students a visual picture of this may help; others may be comfortable with numbers.

5	6
11	

To see if this idea has been understood, ask them to do the same with some larger numbers, e.g. ‘Given that 17 + 5 = 22, can you make another addition and 2 subtraction sentences?’

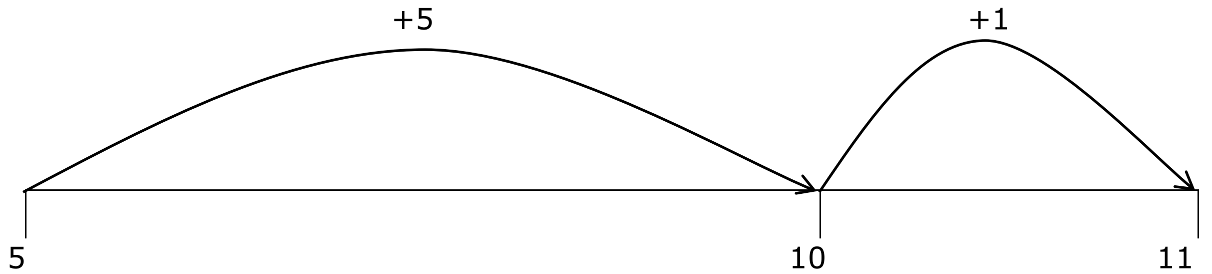
d) Use a number line to show addition and subtraction.

Students will probably have met number lines before, and may have found them helpful, or not, in the past. For those who are unclear about them, it may be best to start with a picture like that above for 5 + 6 = 11. It can be shown that 11 – 5 = 6 and this could be simplified into a number line starting from 5 as it is the part between 5 and 11 that is needed to work out the subtraction.

This leads to a blank number line:

From here we need to find how much is between 5 and 11. Use the language of ‘signposts’ as numbers on the way that can be useful, and stress that these will usually be multiples of 10, and so end in zero. Ask what the signpost would be in this situation. Then write it in, stressing that it doesn’t have to be in an accurate place:

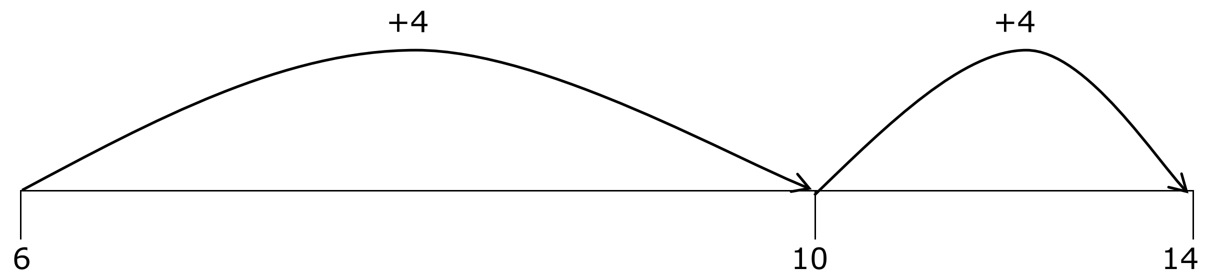
Then we need to find the jumps, firstly between 5 and 10, then between 10 and 11:



The answer is found by adding the jumps together: 5 + 1 = 6, so 11 – 5 = 6. This way of subtracting can be extended to much larger numbers (see Unit 5 Addition and subtraction 2 and Unit 6 Addition and subtraction 3), to decimals (see Unit 11 Fractions, decimals and percentages 2) and time intervals (see Unit 13 Measures 2). It is an invaluable method for those students who have not been able to master vertical subtraction using decomposition.

e) Find missing numbers in addition and subtraction calculations, up to 20.

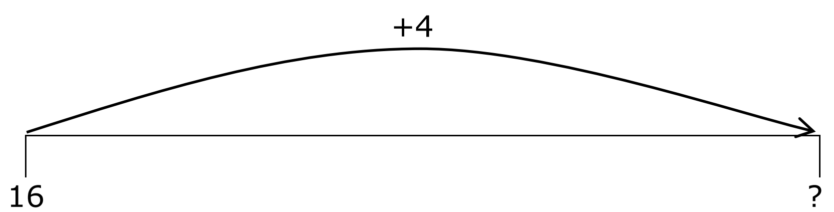
e.g Find the missing number in this calculation: 6 + ☐ = 14
When a student has had the above experience and practice with number lines for subtraction, the following set up should make sense to them:



The missing number is 4 + 4 = 8.
Check by putting it back into the problem: 6 + **8** = 14

e.g. Find the missing number in this subtraction: ☐ – 4 = 16
'We need to find a number which, when we take away 4, the answer is 16. Will the answer be bigger or smaller than 16?' If the student is not sure, or thinks the number will be smaller than 16, they need to try out some numbers which are smaller than 16, such as 12. 12 – 4 = 8, so 12 is too small.
'So the number has to be bigger than 16. In fact, it needs to be 4 bigger.' So the number line could look like this:

Or this:



f) Use knowledge that addition can be done in any order to do mental calculations more efficiently.
Stress finding combinations that add to 10.

g) Use a calculator to add and subtract and to check answers, including using the inverse of subtraction

Keywords

add, plus, sum, total, altogether, count on, count back, double, subtract, minus, take away, difference, addition, subtraction, missing number, inverse, mental calculation, efficient, check, even, odd

Common misconceptions

Has not understood the importance of knowing number bonds, depends on fingers for addition and subtraction.
Stress the importance: start with number bonds to 10 with frequent practice.

Has not understood that addition and subtraction are related.
Objective c) helps to address this.

Thinks that a number line has to be marked in 1s.
Introduce blank number lines for addition and subtraction.

Addition and subtraction 1 check-up

1. Complete this calculation: 14 + 5 =

Write another addition and two subtractions using the same numbers.

2. When an odd number is added to an odd number, is the result odd or even?

Can you explain why?

3. 3 + $\square$ = 20. What goes in the box?

4. What is a good way to add these numbers together: 3 + 8 + 2 + 7?

5. Check your answers to questions 1–4, using a calculator.

6. Check your answer to 20 – 7 = by using the '+' key on a calculator.

Addition and subtraction 1

Opportunities for reasoning/problem solving – teacher notes

Coin investigation 1

You need a collection of 2p and 5p coins – 10 × 2p and 4 × 5p. They can be plastic but it becomes a more real exercise if the coins are genuine.

The activity sheet can be given and the problem discussed so that the student understands what is being asked. There are a couple of examples to start them off.

Answers

All amounts can be made except 1p and 3p.
They would need a 1p coin to make those.
All the amounts over 10p can be made in two ways (because 10p can be made with 5 × 2p or 2 × 5p)
20p can be made in three ways (10 × 2p, 4 × 5p, 5 × 2p + 2 × 5p)
All amounts over 20p can be made (although more coins are needed).

Odd and even investigation

The student could look at the statements and make a guess as to which are correct. They might like to give the characters names. Then they try out some numbers to see which are correct.

Answers

You can make 18 with 3 even numbers or with two odd and one even.

But you can't make 18 with three odd numbers or with two even and one odd, because the result would be odd.

As a supplementary exercise the student could see how many ways they can make the possible totals. This would be a good exercise for working systematically. It could be restricted by saying that all the numbers have to be different. Or it could be made easier by making a lower number, say 10.

A further investigation could be with the same statements, but with an odd number, such as 15. (In this case, the answers are the opposite of the above.)

Coin investigation 1

You have a collection of 2p and 5p coins.

How can you make all the amounts of money up to 20p?

1p	
2p	
3p	
4p	2p, 2p
5p	
6p	
7p	5p, 2p
8p	
9p	
10p	
11p	
12p	
13p	
14p	
15p	
16p	
17p	
18p	
19p	
20p	

Are there some amounts you can't make?
What extra coin would you need to make those?

Which amounts can you make in more than one way?

What do you think about amounts greater than 20p? Do you think you can make them using only 2p and 5p coins?

Odd and even investigation

Investigate whether these statements are true or not.

I can make 18 with three odd numbers.

Jaime says

I can make 18 with three even numbers.

Sunir says

I think I could make 18 with one even and two odd numbers.

Ashley says

I can make 18 with two even numbers and one odd number.

Debbie says

Access Unit 4: Addition and subtraction 1

Objectives

a) **Know by heart all addition and subtraction facts for each number to 10, then 20.**

b) **Understand the effect of adding and subtracting combinations of even and odd numbers.**

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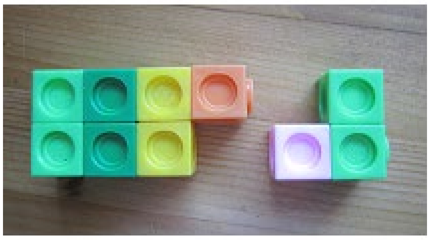

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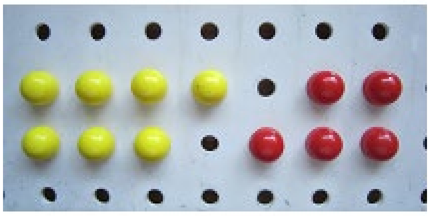

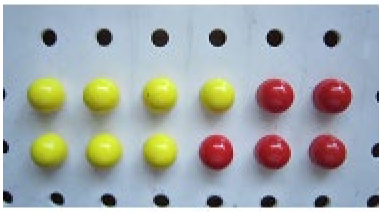

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Then ask for two subtractions using the same numbers. They may need prompting to start with 11. (11 – 5 = 6, 11 – 6 = 5)

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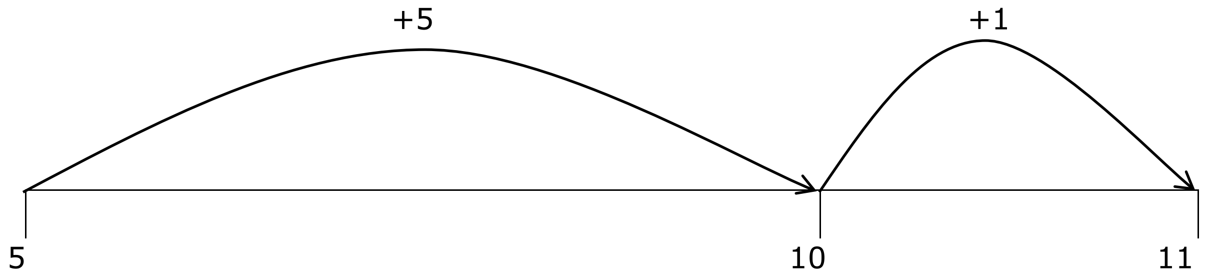

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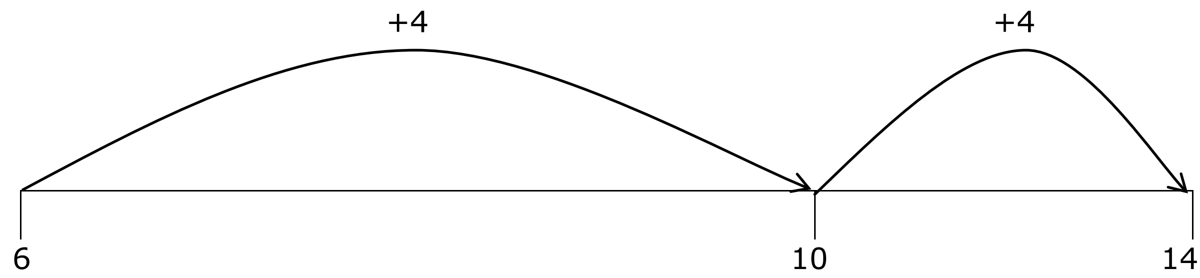

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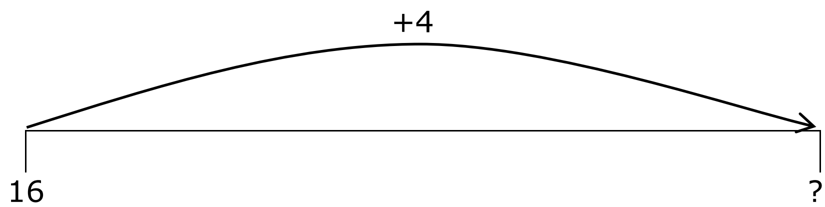

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Addition and subtraction 1

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The student could look at the statements and make a guess as to which are correct. They might like to give the characters names. Then they try out some numbers to see which are correct.

Answers

You can make 18 with 3 even numbers or with two odd and one even.

But you can't make 18 with three odd numbers or with two even and one odd, because the result would be odd.

As a supplementary exercise the student could see how many ways they can make the possible totals. This would be a good exercise for working systematically. It could be restricted by saying that all the numbers have to be different. Or it could be made easier by making a lower number, say 10.

A further investigation could be with the same statements, but with an odd number, such as 15. (In this case, the answers are the opposite of the above.)

Coin investigation 1

You have a collection of 2p and 5p coins.


How can you make all the amounts of money up to 20p?

1p	
2p	
3p	
4p	2p, 2p
5p	
6p	
7p	5p, 2p
8p	
9p	
10p	
11p	
12p	
13p	
14p	
15p	
16p	
17p	
18p	
19p	
20p	

Are there some amounts you can’t make?
What extra coin would you need to make those?

Which amounts can you make in more than one way?

What do you think about amounts greater than 20p? Do you think you can make them using only 2p and 5p coins?

Odd and even investigation

Investigate whether these statements are true or not.

I can make 18 with three odd numbers.

I can make 18 with three even numbers.

I think I could make 18 with one even and two odd numbers.

I can make 18 with two even numbers and one odd number.

Access Unit 5: Addition and subtraction 2

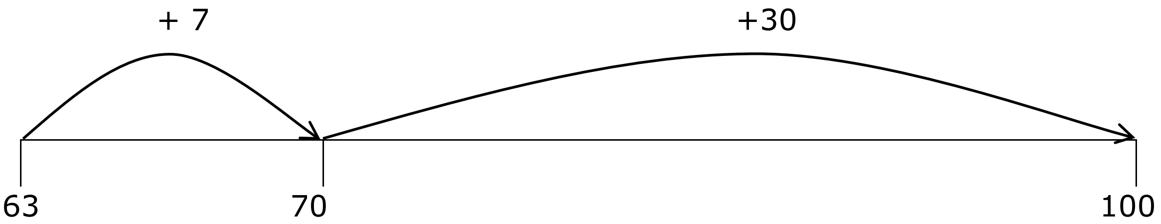
Objectives

a) **Find missing numbers in addition and subtraction calculations up to 100.**

b) **Use knowledge that addition can be done in any order to do mental calculations more efficiently.**

c) **Add and subtract mentally a 'near multiple of 10' to or from a two-digit number, using a compensation strategy.**
Check that they can add and subtract 10 from any number, then use a 100 square to illustrate the relationship between adding/subtracting 10 and 9. Start with adding, then when established extend to +19, +99. Then extend to subtraction.

d) **Find the complement to 100.**
e.g. Find the complement of 63.
Using a number line:



So the complement of 63 is 37.

Using base 10 apparatus:
Make 63

Lay 63 on 100 to show the complement: 37

e) Use known number facts and place value to add or subtract mentally, including any pair of two-digit whole numbers.

f) Add and subtract on a calculator with positive whole numbers.

g) Check subtraction by using the inverse operation on a calculator.

h) Recognise when the answer on the calculator is of the wrong magnitude.

Keywords

add, plus, sum, total, altogether, count on, count back, double, subtract, minus, take away, difference, addition, subtraction, missing number, inverse, mental calculation, efficient, check, even, odd, place value, **near multiple, compensation, complement, positive, magnitude**

Common misconceptions

Thinks that you always have to start with the largest number when adding a string of numbers.

Consistently makes a mistake with finding complements – says that the complement to 100 of 46 is 64.
Use base 10 apparatus or counting on strategies supported by a blank number line to address this.

Has not mastered decomposition for subtraction.
Demonstrate adding on using a blank number line and show how that will work for any subtraction (including decimals).

Addition and subtraction 2 check-up

1. Find the missing numbers.

$56 + \square = 100$

$\square + 9 = 34$

$17 - \square = 12$

2. What is a good way to add these numbers?

$24 + 15 + 62 + 16 + 18 + 5$

I did this on a calculator and the answer was 42.
How do you know that this is wrong (without working it out)?

3. What is a quick way to add 9?
For example, in 67 + 9 =

4. What is a quick way to subtract 9?
For example, in 17 – 9 =

5. Check your answer to 54 – 26 on a calculator by using the ‘+’ key.

Addition and subtraction 2

Opportunities for reasoning/problem solving – teacher notes

Maximum subtraction 1

It may be useful to discuss with the student what will make the biggest number after a subtraction, or you may prefer to have this discussion after the first try and before the second one. They need to be aiming for the biggest possible number at the top and the smallest possible number at the bottom.

Like the Maximum number activity in Unit 2 Place value, ordering and rounding 2, there are elements of luck and strategy here.

Coin investigation 2

Each student will need a set of the coins (1p, 2p, 5p, 10p, 20p, 50p, £1), preferably real ones, but plastic ones are acceptable. They may need support to start, and to be systematic.

Answers

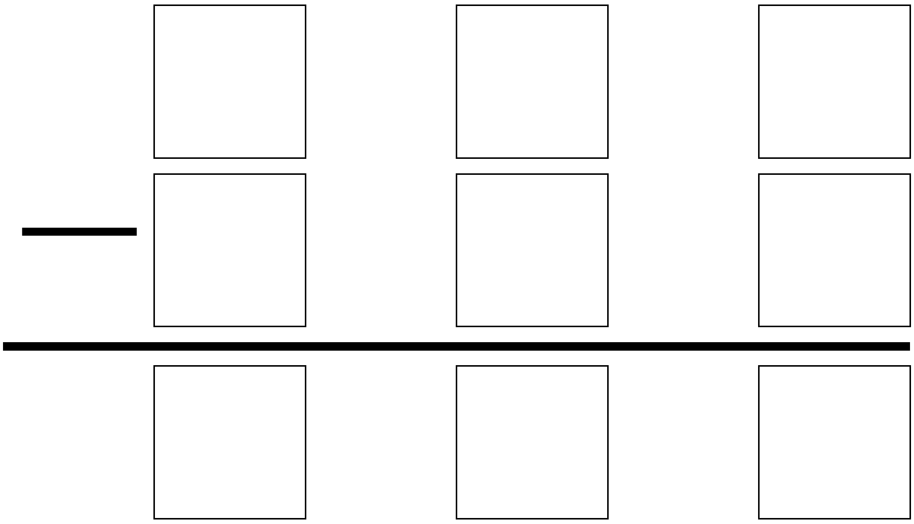
The amounts which can't be made are all those with a 4 or a 9 in them.

The extra coins needed are 1p or 2p and 10p or 20p.

Maximum subtraction 1

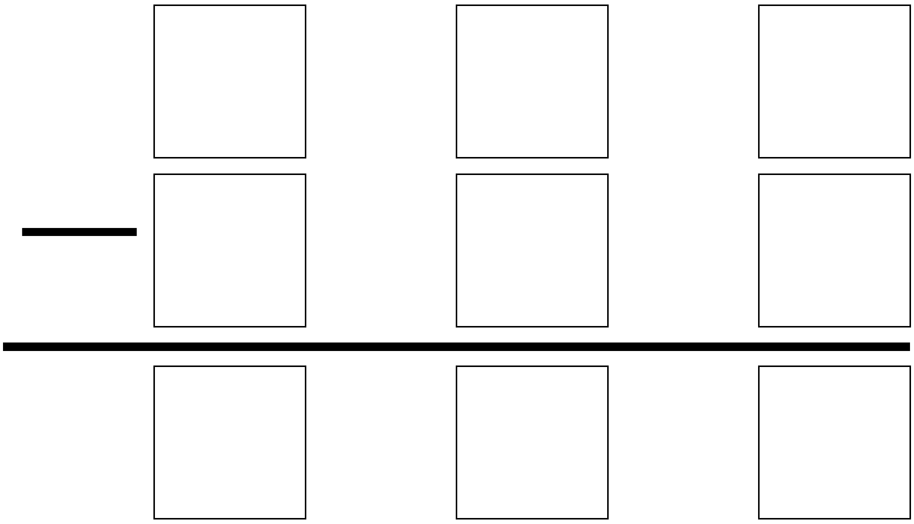
Your teacher will roll a 6-sided dice. You will write each number rolled in one of the boxes. You will make two three-digit numbers and then you are going to subtract the lower one from the higher one. You are trying to make numbers which will give the biggest number possible after the subtraction.

Once you have written a number in a box you cannot change it, so think carefully about the best place to put it.



When you have worked out the result, you can move the numbers to make the biggest possible difference.

Then you can have another go:



Coin investigation 2

You have one of each of these coins: 1p, 2p, 5p, 10p, 20p, 50p, £1.
Shade in which amounts of money you can make with these coins.
Remember you only have one of each!

1p	2p	3p	4p	5p	6p	7p	8p	9p	10p
11p	12p	13p	14p	15p	16p	17p	18p	19p	20p
21p	22p	23p	24p	25p	26p	27p	28p	29p	30p
31p	32p	33p	34p	35p	36p	37p	38p	39p	40p
41p	42p	43p	44p	45p	46p	47p	48p	49p	50p
51p	52p	53p	54p	55p	56p	57p	58p	59p	60p
61p	62p	63p	64p	65p	66p	67p	68p	69p	70p
71p	72p	73p	74p	75p	76p	77p	78p	79p	80p
81p	82p	83p	84p	85p	86p	87p	88p	89p	90p
91p	92p	93p	94p	95p	96p	97p	98p	99p	£1

What do you notice about the amounts you can't make? (They are the ones you haven't shaded in.)

What extra coins do you need to make everything up to £1? You can only have two extra coins.

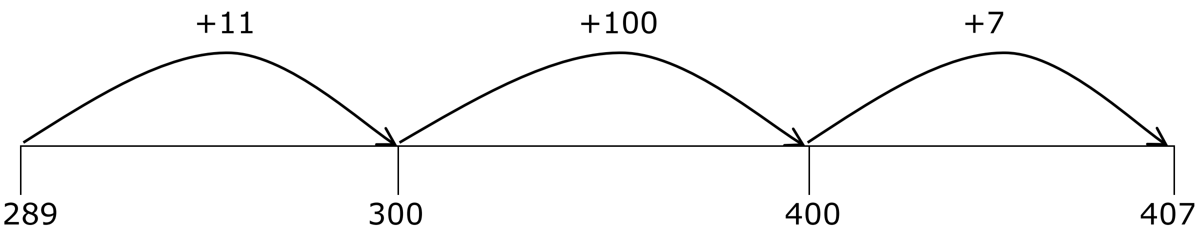
Access Unit 6: Addition and subtraction 3

Objectives

a) Add two integers less than 1000, and more than two integers.

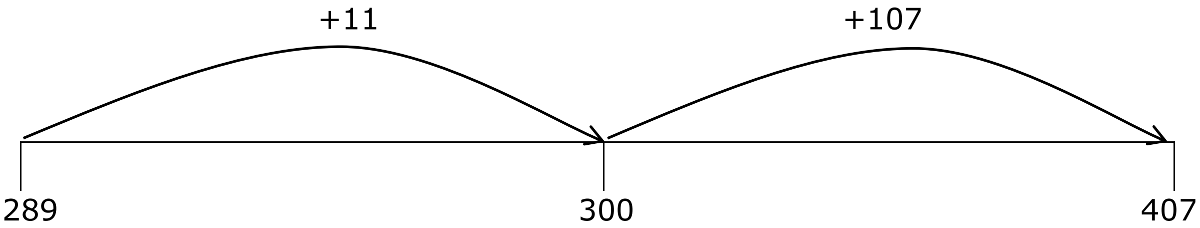
b) Subtract with two integers less than 1000.

Students will have met number lines before (in Unit 4 Addition and subtraction 1 and Unit 5 Addition and subtraction 2 if they have been following this SoW). They may be able to do vertical subtraction using decomposition but may struggle when there is a zero in the number they are subtracting from, e.g. 407 – 289. They may benefit from trying out a number line method for large numbers.



407 – 289 = 11 + 100 + 7 = 118

As they become more efficient, they should be able to do this in just two jumps:



407 – 289 = 11 + 107 = 118

Any three-digit – three-digit subtraction can be done in two jumps: one jump up to the next 100 and then another jump to the larger number.

Note: Some students may confuse this with rounding, so the differences need to be explained.

c) Find missing numbers in addition and subtraction calculations up to 1000.

d) Use a calculator to add and subtract positive whole numbers.

e) Check subtraction by using the inverse operation on a calculator.

f) Recognise when the answer on the calculator is of the wrong magnitude.

Keywords

add, plus, sum, total, altogether, count on, count back, double, subtract, minus, take away, difference, addition, subtraction, missing number, inverse, mental calculation, efficient, check, even, odd, place value, near multiple, compensation, complement, positive, magnitude, integer

Common misconceptions

Thinks that you always have to start with the largest number when adding a string of numbers.

Consistently makes a mistake with finding complements – says that the complement to 100 of 46 is 64.
Use base 10 apparatus or counting on strategies supported by a blank number line to address this.

Has not mastered decomposition for subtraction.
Demonstrate adding on using a blank number line and show how that will work for any subtraction (including decimals).

Addition and subtraction 3 check-up

1. 367 + 874 =

2. 56 + 72 + 74 + 98 =

3. 506 – 238 =

Check your answer on a calculator by using the '+' key.

4. 832 + $\square$ = 1000

5. $\square$ – 67 = 345

I do this last one on a calculator and get 268.
How do you know this is wrong (without working it out)?

Addition and subtraction 3

Opportunities for reasoning/problem solving – teacher notes

Maximum subtraction 2

This is a more difficult version of Maximum subtraction 1 in Unit 5 Addition and subtraction 2.

It may be useful to discuss with the student what will make the biggest number after a subtraction, or you may prefer to have this discussion after the first try and before the second one. They need to be aiming for the biggest possible number at the top and the smallest possible number at the bottom.

Like the Maximum number activity in Unit 2 Place value, ordering and rounding 2, there are elements of luck and strategy here.

Think of a number problems 1

Students may find it helpful to use number lines to explore these problems. They should be encouraged to put their answers back into the problem to check.

Answers

1. 89

2. 73

3. 371

4. 40

5. 100

Word problems using addition and subtraction

These increase in difficulty mainly because of the size of the numbers.

Answers

1. 9

2. Sarah is 77 and Julie is 48.

3. £196

4. £88

5. £391

Maximum subtraction 2

Your teacher will roll a 6-sided dice. You will write each number rolled in one of the boxes. You will make two four-digit numbers and then you are going to subtract the lower one from the higher one. You are trying to make numbers which will give the biggest number possible after the subtraction.

Once you have written a number in a box you cannot change it, so think carefully about the best place to put it.

	☐	☐	☐	☐
−	☐	☐	☐	☐
	☐	☐	☐	☐

When you have worked out the result, you can move the numbers to make the biggest possible difference.

Then you can have another go:

	☐	☐	☐	☐
−	☐	☐	☐	☐
	☐	☐	☐	☐

Think of a number problems 1

1. I think of a number and add 175. The result is 264.
What number did I start with?

2. I think of a number and subtract 46. I have 27 left.
What was the number?

3. I think of a different number and take it away from 500.
I am left with 129. What was this number?

Ready for a challenge?

4. I start with a number and add 58. Then I take the result from 100.
I have 2 left. What was this number?

5. I think of a number and subtract 99, then I take the result from 100.
I have 99 left. What was the number?

Word problems using addition and subtraction

1. I have 24 sweets. 5 of them are red, 10 are green and the rest are pink. How many are pink?

2. Mary is 19, her grandmother Sarah is 58 years older and her mother Julie is 29 years younger than Sarah.
How old are Sarah and Julie?

3. A sofa was priced at £595 but has been reduced to £399.
How much will you save?

4. A table and 4 chairs cost £186.
The table costs £98.
How much are the 4 chairs?

5. I buy the following clothes to go to a wedding:
dress £119
shoes £58
hat £27
jacket £187
How much does my outfit cost altogether?

Access Unit 7: Multiplication and division 1

Objectives

a) Understand the operation of multiplication as repeated addition or as describing an array.

b) Know and use halving as the inverse of doubling.

c) Halve even two-digit numbers with even tens.
Partition numbers, halve and recombine

d) Double numbers up to 50.
Partition numbers, double and recombine.

e) Know by heart facts for the 2 and 10 multiplication tables, then for 5.

f) Know the divisibility rules for 2, 5 and 10.

g) Understand division and recognise that division is the inverse of multiplication.
Start with this picture of a bun tin:

Ask, 'What can you see?' Then ask, 'How many cakes could you cook in this?' Observe what they do, and if they count in ones, ask, 'Is there a quicker way you could find out?' If you are not sure how they counted, ask them. If they respond by saying, 'It's in 3s' or 'It's in 4s', ask them to write down a multiplication sentence to express that.

They should write: $3 \times 4 = 12$ or $4 \times 3 = 12$.

Ask if they can write another multiplication with the same numbers.

Then ask if they can write a division sentence with those numbers. They may need prompting to start with 12. Show them how the picture of the bun tin shows the multiplications and the divisions by pointing out the 3 rows of 4 and 4 columns of 3. Stress that the division sign in 12 ÷ 3 means 'How many 3s are in 12?'

You could move on from here by asking for other ways in which 12 could be arranged, perhaps using pegs and a pegboard. They should come up with a 2 × 6 array. Ask them to write the four number sentences that go with this array.

You can also say that these four numbers (3, 4, 2 and 6) are all factors of 12. Ask them if there is another way to arrange 12. They may need help to find the other pair of factors (1 and 12).

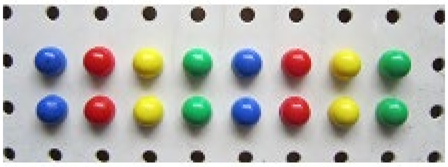
In future lessons a new number could be taken, arrays found and sentences written. The aim is to establish the relationship between multiplication and division and to 'unlock' the meaning of the division sign. The connection between multiplication/division and factor pairs can be reinforced with each different number.

h) Find pairs of factors of numbers up to 20, identifying square and prime numbers.

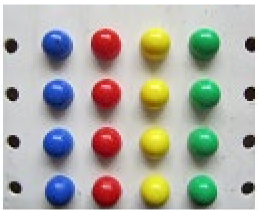
Factors and square numbers

This follows directly from the previous activity. Pegs on a pegboard or linking cubes can be used to support the finding of factors. The student may not realise that square numbers are named because they can be drawn as a square. It can be useful to draw the square numbers on centimetre squared paper.

Start with 16. Ask, 'What goes into 16 exactly?' They should be able to spot that 16 is an even number and therefore 2 is a factor. Suggest they arrange the 16 pegs or cubes into 2s to find out what the corresponding factor is.



Then ask what else might go into 16. If they suggest 3, ask them to rearrange the pegs/cubes into 3s. Continue in this way until they suggest 4 and a square is found. Ask them about the shape.



Explain that this is why 16 is called a square number. At this point it might be appropriate to draw the square numbers up to 25 or 36 on centimetre squared paper and label them, e.g. 4 × 4 = 16, and so on.

Prime numbers

After exploring 16, suggest 17, and ask about the factors. When they realise they can't find any, point out that 1 is a factor of every number, and the number itself is always a factor. The only array that can be made for 17 is 1×17; it only has two factors and is called a prime number. Ask what other numbers they think might be prime and explore the numbers under 10, and on another occasion, numbers up to 20. Stress that 1 is not a prime number because it only has one factor (i.e. itself, 1), and is in fact a square number.

i) Use a calculator to multiply and divide.

Keywords

halve, half, equals, more than, greater than, less than, multiply, array, divide, how many times?, divisibility rule, round, factor, inverse, multiplication, division, prime number, square number, multiple

Common misconceptions

Knows that numbers in the 5× table end in 5 or 0, but does not connect this with recognising which numbers are divisible by 5. Similarly for 2× table (connection with even numbers), and 10× table (numbers which end in 0).

Has not connected doubling and halving with multiplying and dividing by 2.

Has not understood that multiplication and division are related.
Objectives a) and g) support this.

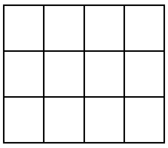
Confuses 'factor' and 'multiple'.

Has not connected 'square' number with the fact that a square can be drawn to illustrate the number.

Thinks of prime numbers as having no factors, rather than just two factors.

Multiplication and division 1 check-up

1. For this array write two multiplications and two division sentences:



Start with $3 \times 4 = 12$.

2. What is half of 42?

3. Double 28.

4. Which of these numbers is divisible by 2, by 5, by 10?

65, 78, 50

	Divisible by 2	**Divisible by 5**	**Divisible by 10**
65			
78			
50			

5. What are the factors of 8, 12, 16, 11?

Which is a square number? Which is a prime number?

6. Check your answers to these calculations, using the calculator.
$4 \times 5 =$

$24 \div 2 =$

Multiplication and division 1

Opportunities for reasoning/problem solving – teacher notes

Chain of numbers

The student may realise that the end of the chain is always 4, 2, 1. They may see that sometimes they have done the work necessary with the previous number (if they are working systematically through the numbers in order).

The shortest chains are for numbers which never lead to an odd number (e.g. 2, 4, 8), and the longest chains are the numbers which follow those with the shortest chains (i.e. 3, 5, 9).

Prime number puzzles 1

It may help the student to make a list of all the prime numbers up to 20 before they start.

Answers

1. 2 + 5 = 7

2. 3 + 7 = 10, 5 + 5 = 10

3. Because all other even numbers have a factor of 2.

4. 3 is 1 more than 2.

5. 2 + 2 = 4, 2 + 7 = 9, 3 + 13 = 16, 5 + 11 = 16

6. No, for example, any odd prime plus 1 will make an even number.

7. No, for example 4 – 1 = 3. It is true for odd prime numbers.

Chain of numbers

You are going to find out what happens when you follow a rule to make a chain of numbers.

The rule is:

If a number is even, halve it.

If it is odd, add 1.

Continue with the new number.

Keep going until the numbers repeat.

Example: Start with 5.

5 is odd, so add 1 ⇨ 6	6 is even, so halve it ⇨ 3
3 is odd, so add 1 ⇨ 4	4 is even, so halve it ⇨ 2
2 is even, so halve it ⇨ 1	1 is odd, so add 1 ⇨ 2

The sequence begins to repeat: 2 ⇨ 1 ⇨ 2 ⇨ 1 and this will go on forever.

See what happens with different numbers less than 10. Which numbers lead to long chains? Which numbers lead to very short chains?

Prime number puzzles 1

All of these puzzles involve prime numbers less than 20.

1. Find two prime numbers which add up to 7.

2. Find pairs of prime numbers which add up to 10.

3. Why is 2 the only even prime number?

4. Which prime number is 1 more than another prime number?

5. Here are some square numbers: 4, 9, 16
Find pairs of prime numbers which add up to make these square numbers.

6. Bob says:

Numbers which are 1 more than square numbers are always prime.

Is he correct?

7. Jill says:

Numbers which are 1 less than square numbers are never prime.

Is she correct?

Access Unit 8: Multiplication and division 2

Objectives

a) Know by heart facts for the 2, 3, 4, 5, 9 and 10 multiplication tables.

9× table using hands:

With palms facing up, put the first finger down for 1×9, the second for 2×9, and so on. Read the fingers on the left of the lowered finger as the number of tens and those to the right as the units.

$2 \times 9 = 18$

$4 \times 9 = 36$

$7 \times 9 = 63$

$9 \times 9 = 81$

This also can be connected with the divisibility rule for 9 – that the digits of multiples of 9 add up to 9. If you have 10 fingers (i.e. including thumbs) and one of them is down, then the rest will add up to 9.

At first students may find this tricky, but with practice they can become very quick, and learn to do it very discreetly under the table.

b) Derive quickly division facts corresponding to the 2, 3, 4, 5, 9 and 10 multiplication tables.

c) Know the divisibility rules for 2, 5, 10, 3 and 9.

d) Find remainders after division.

e) Halve even two-digit numbers with odd tens.

Partition numbers into even tens + 10, halve and recombine.

f) Double any two-digit number.
Partition numbers, double and recombine.

g) Find pairs of factors of numbers up to 50, then 100, identifying square and prime numbers.
Use the divisibility rules.

h) Multiply a three-digit by a one-digit integer.
e.g. 126 × 8

Grid method

Partition the number into hundreds, tens and units:

×	100	20	6
8			

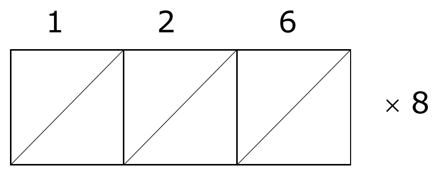
Each number is multiplied by 8 and the products added together:

×	100	20	6
8	800	160	48

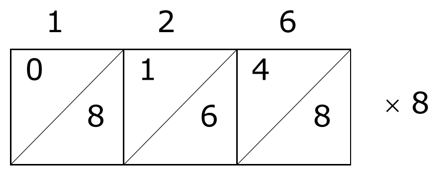
800 + 160 + 48 = 1008

The student may need practice with multiplying multiples of 10.

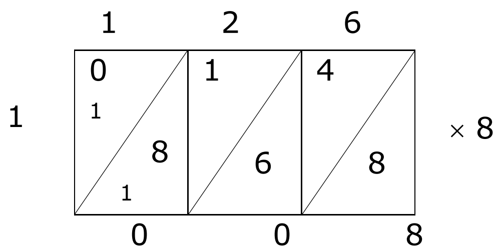
Napier's bones



Each number is multiplied by 8 and the tens and units of the answers are written in the cells separated by the diagonal lines:



Then add along the diagonals, with any tens carried into the next diagonal:



Answer is given by the numbers outside the grid: 1008.

i) Divide a three-digit by a one-digit integer.

Some students may be confident with the 'bus stop' method of short division, but others may have not understood division at all in the past. The 'chunking' method provides another way to approach division – from multiplication.

e.g. $147 \div 8$

Ask, 'What does this mean?' In order to understand division, they need to know that it means 'How many 8s are there in 147?'

This is about the 8× table, so we start with:	$10 \times 8 = 80$
Then we can double this:	$20 \times 8 = 160$, but this is too big
We can halve 10×8:	$5 \times 8 = 40$
We can add 10×8 and 5×8:	$15 \times 8 = 40 + 80 = 120$
$147 - 120 = 27$, so we have 27 left:	$3 \times 8 = 24$
So, adding 15×8 and 3 x 8	$18 \times 8 = 120 + 24 = 144$

$147 - 144 = 3$ so we have 3 left which is the remainder.

Answer is 18 r 3.

j) Use a calculator to multiply and divide with positive whole numbers.

k) Check division by using the inverse operation on a calculator, and extend to remainders.

To check the answer to the division above, ask them to enter $147 \div 8$ into the calculator. The answer will come up as 18.375, so this tells us that the 18 is right, but does not confirm our remainder. To do that we have to reverse the calculation, starting from our answer so we enter $18 \times 8 + 3$ into the calculator, and if our answer is correct, the calculator will display the starting number 147.

l) Recognise when the answer on the calculator is of the wrong magnitude.

Keywords

halve, half, equals, more than, greater than, less than, multiply, array, divide, how many times?, divisibility rule, round, factor, inverse, multiplication, division, square number, prime number, **remainder, near multiple, hundreds, magnitude, chunking, target number, grid method, Napier's bones, magnitude, partition, positive**

Common misconceptions

Has not been shown divisibility rules for 3 and 9, by adding digits.

Has difficulty in drawing grid for Napier's bones.

Uses the grid method, but does not partition the numbers correctly.

Does not understand division as 'how many ... go into ...'
Ask the question, 'What does this mean?' before any division. Identifying the answer to the division from the working can be a problem when starting chunking. Going back to the original calculation can help.

Has not connected 'square' number with the fact that a square can be drawn to illustrate the number.

Thinks of prime numbers as having no factors, rather than only two factors.

Multiplication and division 2 check-up

1. Halve 74.

2. Double 87.

3. Find all the factors of 45, 24, 16, 31.

Identify the square number and the prime number.

4. $124 \times 6 =$

5. $125 \div 5 =$

6. $56 \div 3 =$

Check your answers to questions 1–6 on a calculator.

7. I did $452 \div 24$ on a calculator and got the answer 125.
How do you know this is wrong (without working it out)?

Multiplication and division 2

Opportunities for reasoning/problem solving – teacher notes

Grid for multiplication and division facts

The only multiplication tables used here are those targeted in this unit, that is, 2, 3, 4, 5, 9 and 10.

If students find it difficult to get started on the first one, direct them to 30 and ask what 10 could be multiplied by to get 30. Similar questions could be asked about 20, 36 and 45. Once they have found the multipliers, the rest is simply an exercise in working out the table facts.

The second one is more difficult because no multipliers are given. If they start with 15, then the only two possible numbers are 3 and 5. Looking at the 27 or the 20 determines that the 3 is on the side and the 5 on the top.

Answers

×	**2**	5	**4**	**3**	9
10	**20**	**50**	**40**	30	**90**
4	**8**	20	**16**	**12**	**36**
3	6	**15**	**12**	**9**	**27**
9	**18**	**45**	36	**27**	**81**
5	**10**	**25**	**20**	**15**	45

×	**3**	**5**	**9**	**4**
3	**9**	15	27	**12**
9	**27**	**45**	**81**	36
2	6	**10**	18	**8**
4	**12**	20	**36**	**16**

Prime number puzzles 2

Before tackling these puzzles it may help the student if they complete the Sieve of Eratosthenes, using a 100 square with the 1 crossed out; they cross off multiples of 2, 3, 5, 7, except for the first multiple. This will give them all the prime numbers under 100.

Answers

1. 2 + 47

2. 2 and any other two primes

3. 3, 5, 11, 17

4. 1, 4, 16, 36

5. Because odd squares +1 make an even number, and 2 is the only even prime.

6. 31 + 19, 37 + 13

7. 43 + 7, 47 + 3

Calculator puzzles

This activity is designed to get students to apply their knowledge of the connections between operations.

Answers

(other answers may be possible)

1. 146 + 146

2. 96 ÷ 2 ÷ 2

3. 72 ÷ 5 ÷ 2

4. 67 + 67 + 67

5. 84 × 10 ÷ 2

6. 57 × 10 – 57

Grid for multiplication and division facts

Find the missing numbers in this multiplication grid:

×		5			9
10				30	
		20			
3	6				
9			36		
					45

Now try this one – it's a bit more tricky!

×				
		15	27	
				36
	6		18	
		20		

Prime number puzzles 2

All these puzzles involve prime numbers under 100.

1. Find two prime numbers which add together to make 49.

2. Find three prime numbers which add together to make an even number.

3. There are four prime numbers under 20 which, when you add 2, you get another prime number. Can you find them?

4. Which square numbers under 100 are one less than a prime number?

5. Why is there only one odd square number in your answer to question 4?

6. Find one prime number between 30 and 40 and another prime number between 10 and 20 which add together to make 50.

Can you find another pair of numbers?

7. Find two pairs of prime numbers which satisfy this condition:
One number is over 40, the other under 10, and they add together to make 50.

Calculator puzzles

In all these puzzles there is something wrong with your calculator – one or two keys don't work each time.
Can you get your calculator to do what you want anyway?
Write down what you would key into the calculator, and the answer it gives you.

1. The key for number 2 does not work.
How can you work out 146×2 on the calculator?

2. The key for number 4 does not work.
How can you work out $96 \div 4$?

3. The keys for 1 and 0 are not working.
How can you calculate $72 \div 10$?

4. The $\times$ key is not working.
How can you work out 67×3?

5. The key for number 5 and the + key are broken.
How can you work out 84×5?

6. The key for number 9 is not working.
How can you work out 57×9?

Access Unit 9: Multiplication and division 3

Objectives

a) Know by heart all multiplication facts up to 10 × 10.

b) Alternatively, know the square numbers up to 100 and use them to derive the 'difficult' times table facts.

Multiplication grid with square numbers shaded.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Most students will be able to recall facts for numbers up to 25. And most will know 10 × 10 = 100.

Helpful hints:

6 × 6 = 36 (Lots of 6s)

7 × 7 = 49 (Wakey wakey, rise and shine, seven sevens are 49)

8 × 8 = 64 (I ate and I ate and I was sick on the floor, eight eights are 64)

9 × 9 = 81 (From the 9× table finger multiplication)

6 × 7 is 6 × 6 add another 6, or 7 × 7 less a 7. Some students may prefer to work from their 5× table if this is secure. 5 × 7 = 35 add another 7

7 × 8 is 7 × 7 add 7

6 × 8 is 3 × 8 doubled or do as 6 × 6 add 2 × 6 (it is easier to add 12 than to add 6 and then add another 6)

c) Multiply a two-digit by a two-digit integer.

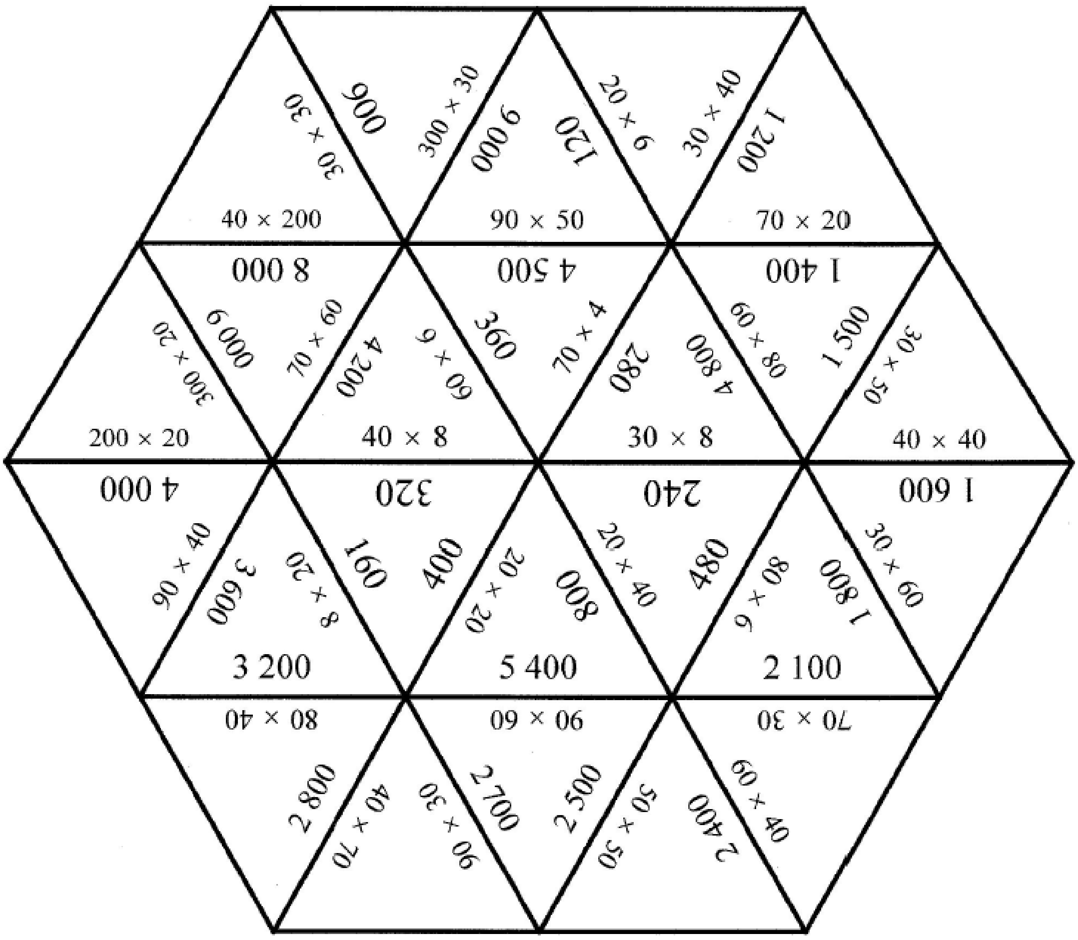
d) Derive quickly division facts corresponding to multiplication tables up to 10 × 10.

e) Multiply a three-digit by a two-digit integer.

See the three pages of jigsaw pieces supplied for practice with this (after Common misconceptions). For best results, laminate them before cutting out the triangles.

The aim is to arrange the triangles so that a multiplication on one side of a triangle (e.g. 20 × 6) is adjacent to the answer on the side of another triangle (i.e. 120).

Solution



f) Divide a three-digit by a two-digit integer.

Extend chunking to larger numbers.

g) Use a calculator to multiply and divide positive whole numbers.

h) Check division by using the inverse operation on a calculator, and extend to remainders.

i) Recognise when the answer on the calculator is of the wrong magnitude.

Keywords

halve, half, equals, more than, greater than, less than, multiply, array, divide, how many times?, divisibility rule, round, factor, inverse, multiplication, division, remainder, near multiple, hundreds, magnitude, chunking, target number, grid method, Napier's bones, magnitude, partition, positive, square number, prime number, **derive**

Common misconceptions

Has difficulty with place value when using grid method for multiplication.

Has not been shown divisibility rules for 3 and 9, by adding digits.

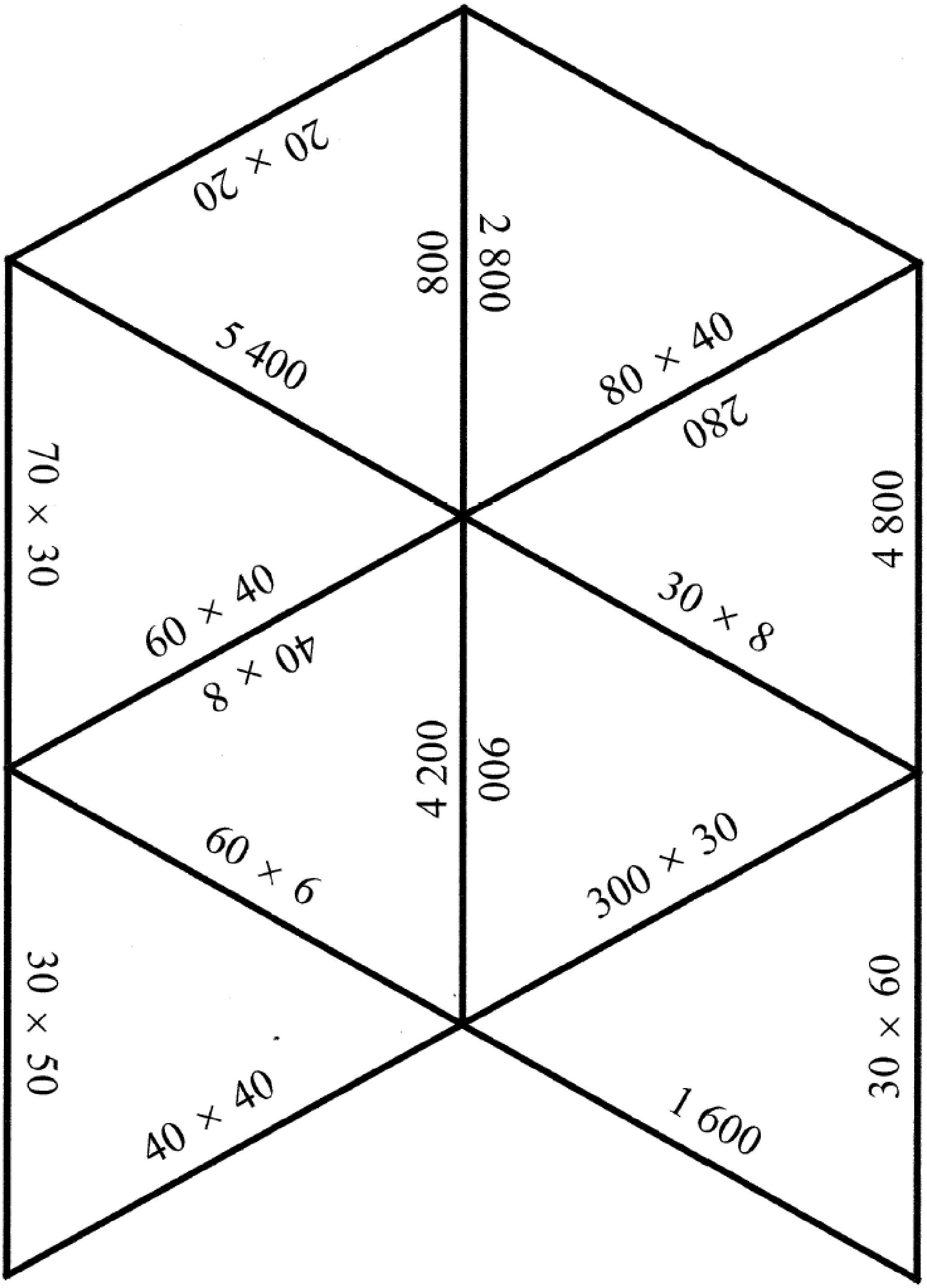
Has difficulty in drawing grid for Napier's bones.

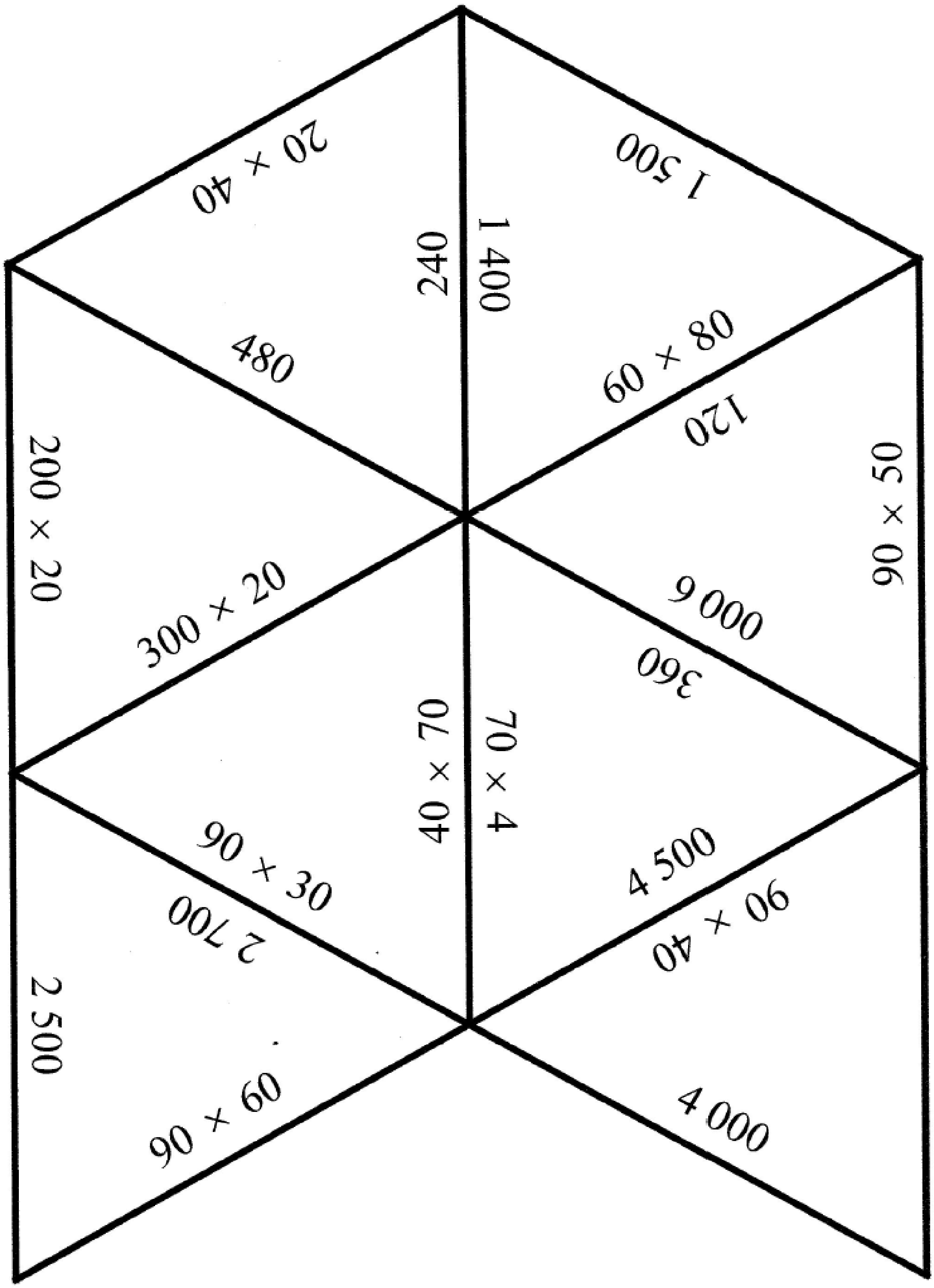
Uses the grid method, but does not partition the numbers correctly.

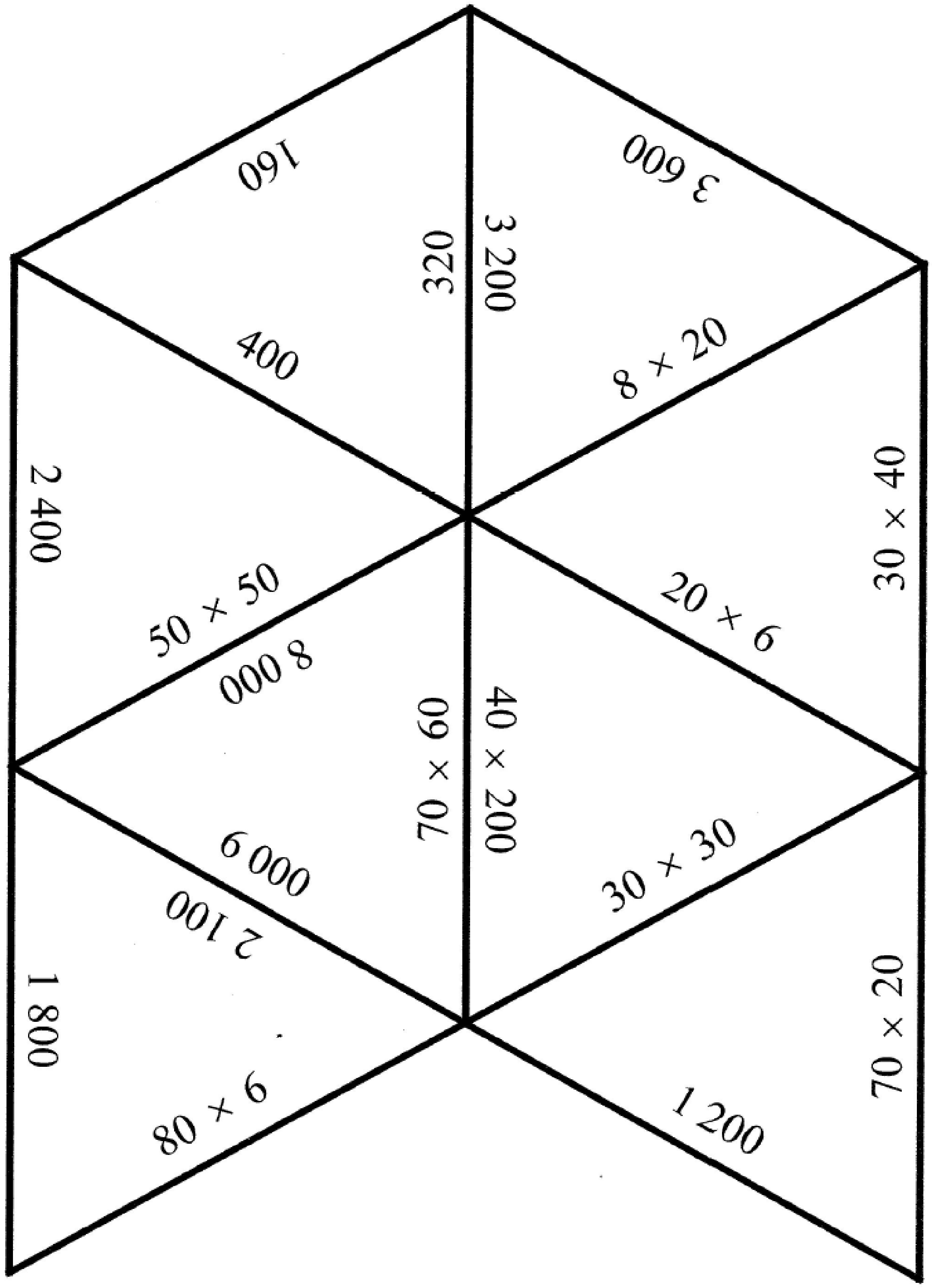
Does not understand division as 'how many ... go into ...'
Ask the question, 'What does this mean?' before any division. Identifying the answer to the division from the working can be a problem when starting chunking. Going back to the original calculation can help.

Has not connected 'square' number with the fact that a square can be drawn to illustrate the number.

Thinks of prime numbers as having no factors, rather than only two factors.


20 × 20
800
2 800
5 400
80 × 40
280
70 × 30
4 800
60 × 40
40 × 8
30 × 8
4 200
900
60 × 6
300 × 30
30 × 50
30 × 60
40 × 40
1 600


20 × 40
1 500
240
1 400
480
60 × 80
120
200 × 20
90 × 50
300 × 20
9 000
360
40 × 70
70 × 4
90 × 30
2 700
4 500
90 × 40
2 500
90 × 60
4 000


160
3 600
320
3 200
400
8 × 20
2 400
30 × 40
50 × 50
20 × 6
8 000
70 × 60
40 × 200
6 000
30 × 30
2 100
1 800
70 × 20
80 × 6
1 200

Multiplication and division 3 check-up

1. $23 \times 67 =$

2. $762 \times 28 =$

3. $456 \div 13 =$

Then check your answers to questions 1–3 on a calculator using the inverse operation.

4. I do 567×42 on a calculator and get the answer 2314.
How do you know this is wrong (without working it out)?

Multiplication and division 3

Opportunities for reasoning/problem solving – teacher notes

Think of a number problems 2

Depending on the student's previous experience with this sort of puzzle, they may need guidance on using the inverse operations to work out the starting number. Questions 4 and 5 test their ability to do this on the calculator. They should be encouraged to put their answers back into the problem to check.

Answers

1. 7

2. 49

3. 6

4. 13

5. 552

Word problems using multiplication and division

Some students may still need to use a calculator in order to work out these problems.

Answers

1. **a.** 90 **b.** 5

2. 4

3. 312

4. 21

5. 132

6. 272

Think of a number problems 2

1. I think of a number, multiply it by 4 and subtract 9. The result is 19.
What was my starting number?

2. I think of a number and divide it by 7. I add 72 and the result is 79.
What was my starting number?

3. I start with a number and multiply it by 12. Then I subtract 4 and divide it by 2. I end up with 34.
What number did I start with?

Try these with your calculator.

4. I think of a number, add 47 and multiply by 26. The result is 1560.
What number did I start with?

5. I subtract 272 from a number and divide the result by 14. I end up with 20.
What was my starting number?

Word problems using multiplication and division

1. I buy 15 packets of mini-chocolate bars for a party. There are 6 bars in each pack.

a. How many bars are there altogether?

b. If 18 people are at the party, how many bars will each person get?

2. I buy a big bag of sweets for the party. The bag contains 72 sweets. How many sweets will each person get?

3. I am building a brick wall. Each layer of the wall needs 52 bricks. I want to build the wall 6 bricks high. How many bricks will I need to do this?

4. I plant some carrots in rows of 12. There are 252 carrots altogether. How many rows do I plant?

5. Around the carrots I plant some rows of leeks because they help to keep carrot fly away. I need one more row of leeks than of carrots – can you see why? I plant 6 leeks in each row. How many leeks is that?

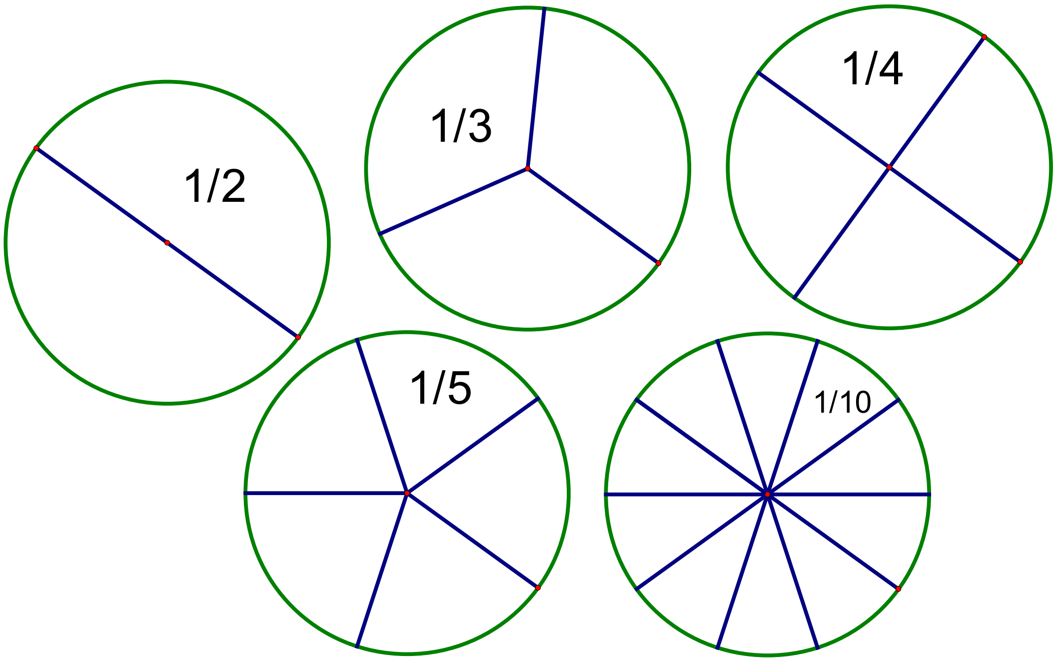
6. I also planted 17 rows of lettuces with 16 in each row. How many lettuces did I plant altogether?

Access Unit 10: Fractions, decimals and percentages 1

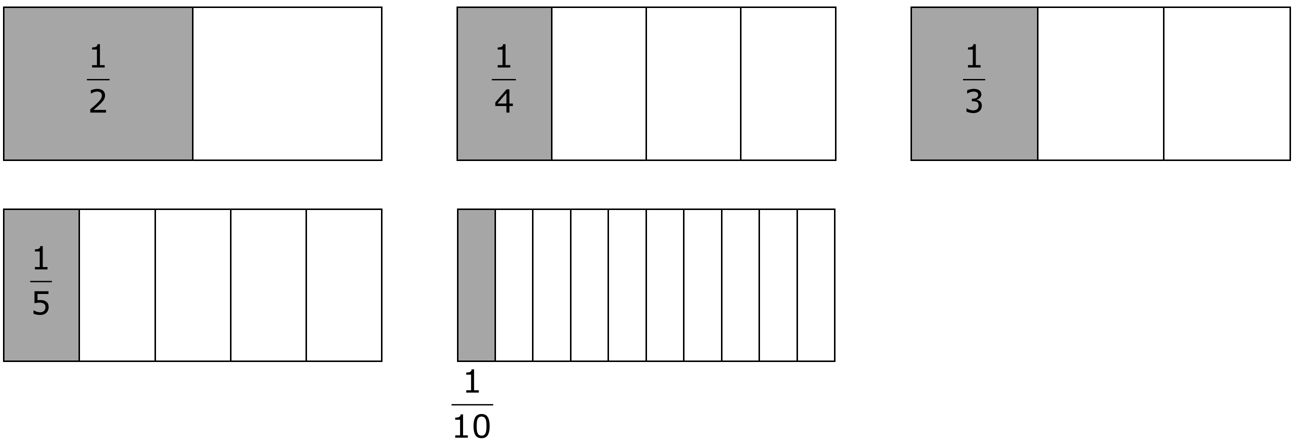
Objectives

a) Recognise unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, and use them to find fractions of shapes and numbers.

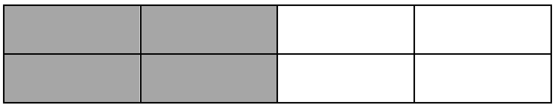
These diagrams can be related to pizza. 'Would you prefer $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{10}$ of a pizza?'



These can be related to bars of chocolate.



b) Recognise the equivalence of simple fractions.



$\frac{1}{2}$ is equivalent to $\frac{2}{4}$ (reading each column as $\frac{1}{4}$) and $\frac{4}{8}$ (reading each cell as $\frac{1}{8}$).

$\frac{1}{2}$ is equivalent to $\frac{3}{6}$

$\frac{1}{3}$ is equivalent to $\frac{2}{6}$

c) Place fractions and decimals on a number line, recognise simple equivalents (0.5 & $\frac{1}{2}$, 0.1 & $\frac{1}{10}$, 0.25 & $\frac{1}{4}$, 0.75 & $\frac{3}{4}$).

Start with a 0–1 number line and extend. Use parallel number lines to show equivalence of fractions and decimals (this is extended in FDP2).

Keywords

fraction, unit fraction, numerator, denominator, half, third, quarter, fifth, tenth, equivalent, decimal, equal parts, decimal point, number line, whole

Common misconceptions

Thinks that the bigger the denominator, the bigger the fraction.

This can be addressed by showing fractions of diagrams, e.g. show $\frac{1}{3}$ *and* $\frac{1}{8}$ *of the same shape.*

Fractions, decimals and percentages 1 check-up

1. Put these unit fractions in order from smallest to largest.

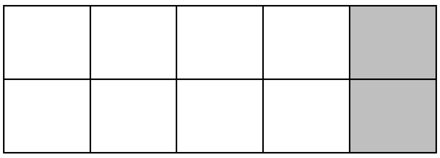
$\frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{1}{10}, \frac{1}{8}$

2. **a.** What is $\frac{1}{4}$ of 12?

b. What is $\frac{1}{10}$ of 50?

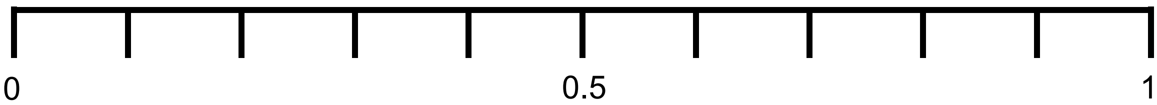
3. Shade $\frac{1}{3}$ of this shape.

4. What fraction of this shape is shaded?

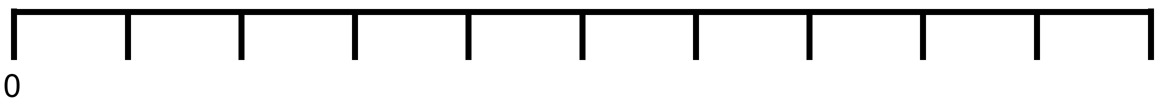


5. Write the missing decimals and fractions on these number lines.

Decimal



Fraction



Where do $\frac{1}{4}$ and $\frac{3}{4}$ go?

Fractions, decimals and percentages 1

Opportunities for reasoning/problem solving – teacher notes

Word problems with fractions

Answers

1. Red 6, yellow 4, pink 2

2. No, because a bigger denominator means that it has been split into more pieces which are therefore smaller.

3. Yes

4. $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$

5. Yes

6. No, 0.1 is the same as $\frac{1}{10}$.

Making fractions and comparing with $\frac{1}{2}$

Students may need support in identifying which number line to put the $\frac{1}{4}$s and the $\frac{1}{3}$s on. A useful supplementary question could be: 'If I have $\frac{1}{4}$ of a cake and you have the rest, what fraction will you have?'

Word problems with fractions

1. I have 12 sweets. $\frac{1}{2}$ of them are red and $\frac{1}{3}$ are yellow.

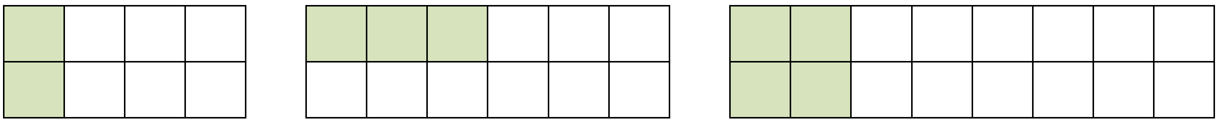
The rest are pink.

How many are: red ____ yellow ____ pink ____

2. Jamie says: $\frac{1}{4}$ of a pizza is more than $\frac{1}{3}$ because 4 is bigger than 3. Is he correct?

3. Ranjit says: I like carrot cake. $\frac{2}{4}$ of a cake is the same as $\frac{1}{2}$. Is she correct?

4. Use these diagrams to find fractions which are equivalent to $\frac{1}{4}$.



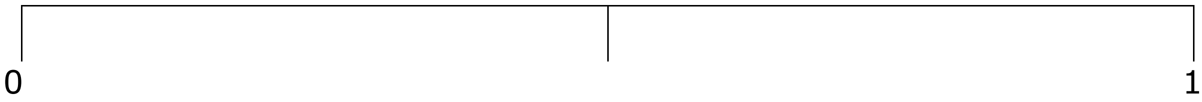
5. Pal says: $\frac{1}{2}$ is the same as 0.5, $\frac{1}{4}$ is the same as 0.25, and $\frac{3}{4}$ is the same as 0.75. Is she correct?

6. Omar says: I think 0.1 is the same as $\frac{1}{1}$. Is he correct?

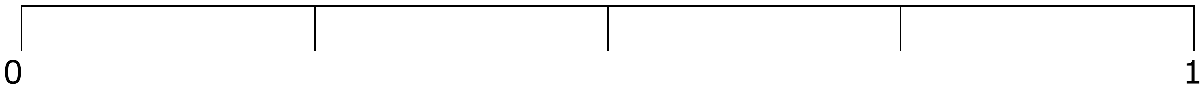
Making fractions and comparing with $\frac{1}{2}$

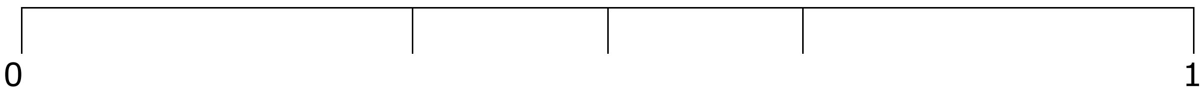
You are going to make some fractions and place them on these number lines.

Start by marking $\frac{1}{2}$ on each line.



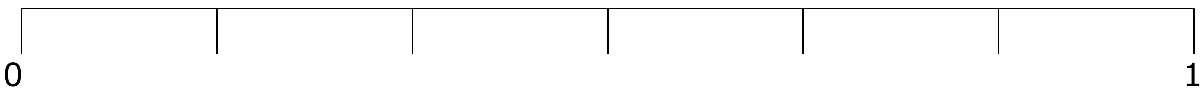
Using the numbers 1, 2, 3 and 4, make some fractions with denominators of 4 or 3 and place them on these lines:





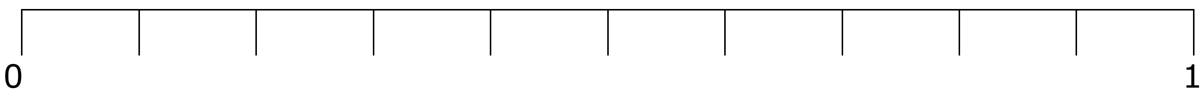
Which fractions are bigger than $\frac{1}{2}$?

Now use the numbers 1, 2, 3, 4, 5 and 6 to make some fractions with a denominator of 6 and place them on this line.



Which fractions are bigger than $\frac{1}{2}$?

Make some fractions with 10 as the denominator and place them on this line. Write the decimals which go with them as well.



Access Unit 11: Fractions, decimals and percentages 2

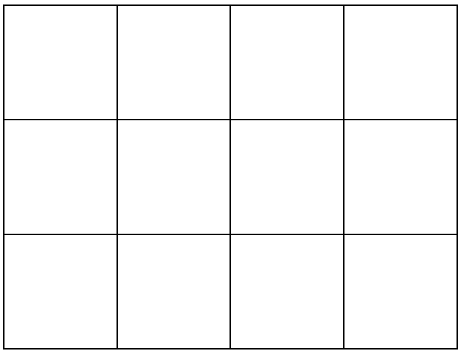
Objectives

a) Recognise simple fractions that are several parts of a whole, and be able to shade shapes to illustrate those fractions.

b) Find fractional quantities of numbers up to 20 (e.g. $\frac{2}{3}$ of 15, $\frac{5}{6}$ of 18).

c) Recognise the equivalence of simple fractions.

This is best done with an A4 sheet marked into $\frac{1}{12}$s as a 3×4 array.



First establish that 1 cell is $\frac{1}{12}$. Then ask the student to fold the paper to show $\frac{1}{2}$ of the whole sheet. Ask how many $\frac{1}{12}$s this is. 'So, $\frac{1}{2}$ is the same as $\frac{6}{12}$.'

Remind the student that we call fractions like these equivalent fractions.

Continue by asking them to show $\frac{1}{4}$ of the sheet ($\frac{3}{12}$), $\frac{1}{3}$($\frac{4}{12}$) and $\frac{1}{6}$ ($\frac{2}{12}$).

Some students may find it difficult to see how to fold the paper to get $\frac{1}{3}$ and $\frac{1}{4}$ and may need to be supported by asking, 'If this were chocolate and you were going to split it between 4 (or 3) of you, what would be the easiest way to break it?'

Students who need more reinforcement could draw all these out on centimetre. squared paper and label them appropriately.

Different arrays could be used to explore other equivalents, e.g. 4×4 to explore quarters, eighths and sixteenths.

d) Place fractions and decimals on a number line.

Cut strips from A3 paper. Give the student one strip and ask them to fold it into half and label the fold line $\frac{1}{2}$. Then they fold it into quarters and label $\frac{1}{4}$ and $\frac{3}{4}$. Continue to find and mark eighths.

Take another strip and fold into thirds. Discuss why this is much more difficult than the previous fractions. By folding approximately and adjusting, find and mark the $\frac{1}{3}$s, and then the $\frac{1}{6}$s. Ask, 'Finding the thirds was difficult. Why is it now easy to find the sixths?' Continue with this strip, finding and marking the $\frac{1}{12}$s.

Put the strips together, matching up the ends. Now all the fractions which have been marked can be compared.

Ask questions such as:

- Which is bigger, $\frac{2}{3}$ or $\frac{3}{4}$?
- Which is bigger, $\frac{3}{8}$ or $\frac{2}{6}$?
- Find six fractions which are smaller than $\frac{1}{2}$.
- Find two fractions which are smaller than $\frac{1}{4}$.
- Which fractions are bigger than $\frac{2}{3}$?
- What equivalents to $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$ can you find?
- What equivalents to $\frac{1}{3}$ and $\frac{2}{3}$ can you find?

e) Use decimal notation for tenths and hundredths, including money and measurement.

Use measurement in centimetres as a practical context for 1 decimal place, and money for 2 decimal places.

f) Relate fractions to division and to their decimal representations.

g) Understand percentage as the number of parts in every 100, and find simple percentages of small whole-number quantities.

Find 10% of a number by dividing by 10 (because $10 \times 10\% = 100\%$), halve to find 5%, double for 20%. Understand that 25% is equivalent to $\frac{1}{4}$ and 50% to $\frac{1}{2}$. Find 1% by dividing by 100 (reinforcing objective **i)** below).

h) Interpret a calculator display as money.

i) Use the calculator to explore the effect of multiplying and dividing by powers of ten.

j) Use a calculator to add and subtract decimals.

Extend number line strategy to decimal subtraction.

k) Use a calculator to convert fractions to decimals.

Keywords

fraction, unit fraction, numerator, denominator, half, third, quarter, fifth, tenth, equivalent, decimal, equal parts, decimal point, number line, whole, **percentage**, **parts in a hundred**, **hundredths**, **powers of ten**, **convert**

Common misconceptions

Reads 0.17 as 'nought point seventeen' or 'Oh point one seven'.

Thinks that multiplying by 10 means adding a zero to the end of the decimal number.

Use a calculator to show that 4.50 is the same as 4.5.

Knows that to find 10% you divide by 10 and thinks that to find 20% you divide by 20.

Fractions, decimals and percentages 2 check-up

1. Shade this diagram to show $\frac{4}{5}$.

What equivalent fraction is shown in this diagram?

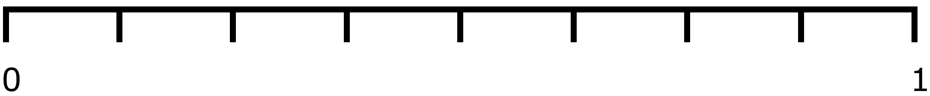
2. Find three equivalent fractions for $\frac{2}{3}$.

3. What is $\frac{3}{4}$ of 20? $\frac{2}{5}$ of 25? $\frac{5}{6}$ of 24?

4. Place these fractions on the number line:

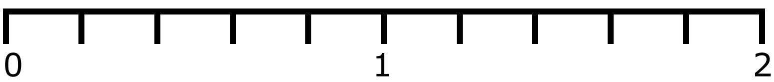
$\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{8}, \frac{5}{8}, \frac{1}{8}, \frac{4}{8}, \frac{2}{8}, \frac{6}{8}$

What equivalent fractions are here?



5. Place these decimal numbers on the number line:

0.5, 1.5, 2.5, 1.75, 2.25, 0.75



Write the numbers in fractions underneath e.g. $1\frac{1}{2}$

6. Write 145p as £.

7. On a calculator work out £4.85 + £3.05 and write your answer on paper.

8. Use a calculator to find the decimal equivalents of these fractions.

$$\frac{1}{4}, \frac{1}{8}, \frac{2}{5}, \frac{2}{3}$$

9. What is:
10% of 40?

25% of 16?

75% of 20?

40% of 30?

5% of 40?

10. What is:
0.4×10?

$56 \div 10$?

7.2×100?

$35 \div 100$?

Check on your calculator.

Fractions, decimals and percentages 2

Opportunities for reasoning/problem solving – teacher notes

Money problems

The student may need reminding about how to interpret the display on their calculator. They may also need support in converting amounts given in pence to the decimal equivalent, or in converting pounds to pence. They should be encouraged to work with pounds as this reinforces working with decimals.

Answers

1. £4.10

2. 10p

3. £1.20

4. Brenda’s Café £8.20, Corner Café £7.60

5. Corner cafe – roll and crisps

Problems involving fractions

Answers

1. Jamil is correct.

2. Katy is wrong: $\frac{2}{5}$ of 20 (8) is less than $\frac{3}{4}$ of 16 (12).

3. Rael is wrong: $\frac{3}{10}$ of 20 (6) is the same as $\frac{1}{3}$ of 18 (6).

4. Sara is correct.

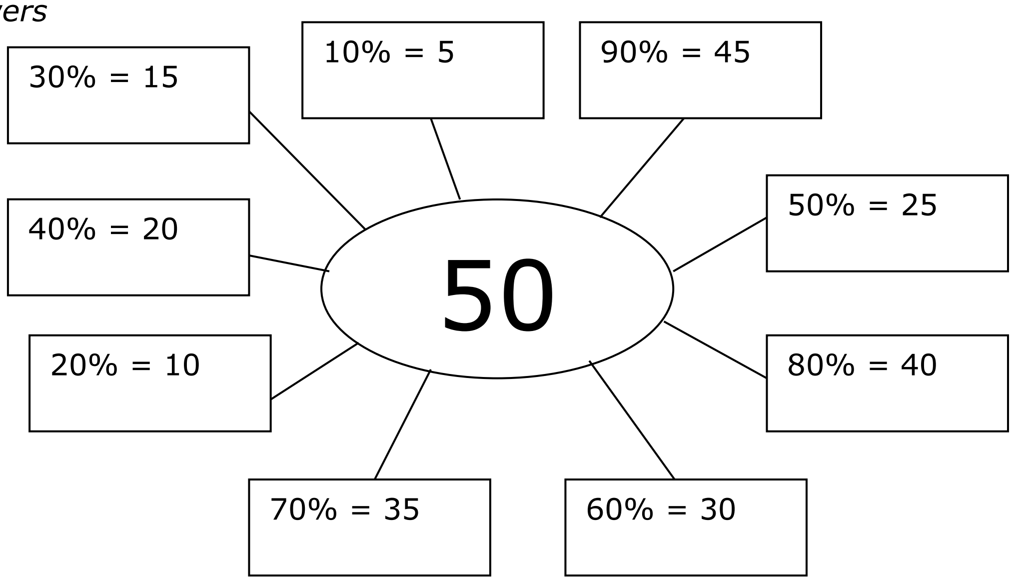
5. Any correct sentences replacing statements 2 and 3.

Simple percentage problems

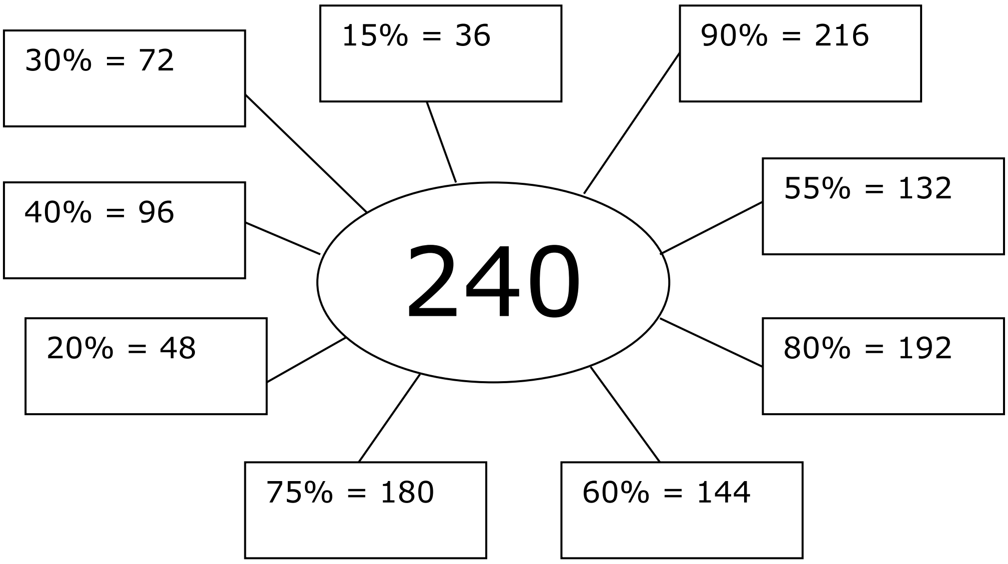
Encourage the student to find 10% first and the rest follow.

Answers

1.



2. What is 10% of 240?(24); 5%? (12, by halving 10%); 50%? (120); 25%? (60, by halving 50%). The rest can be worked out by multiplying or adding. Some students may need a calculator for 80% and 90%.



Decimal problems using a calculator

The student may need help with deciding how to enter the numbers into a calculator and which operation to use. Encourage them to draw diagrams to illustrate the problems.

Answers

1. Longer by 2.7 cm

2. Yes, 0.4 m

3. 1.3 km

4. Three shelves. Cut 0.2 m off the first piece (1.6 m), and cut two lengths of 1.4 m from the second (2.9 m), leaving 0.1 m.

Money problems

Use your calculator to solve these problems. Remember to give your answers in money format (£.p).

1. On Monday I spend £2.50 on a sandwich, 55p on a bag of crisps and £1.05 on a drink at Brenda's Cafe. How much do I spend altogether?

2. On Tuesday I notice that there is a special offer:

Brenda's special Tuesday offer

sandwich + crisps + drink £4

How much will I save if I have the same lunch as Monday?

3. On Wednesday in the Corner Cafe I pay £2.45 for a roll, 45p for crisps and 90p for a drink. How much change will I get from £5?

4. On Thursday a friend offers to buy me lunch. If we both have the same lunch, how much will it cost for both lunches if we go to Brenda's Cafe? (The special offer has finished.) How much will it cost if we go to the Corner Cafe?

5. On Friday I only have £3 left to buy my lunch. Where should I go and what can I get for my money?

Problems involving fractions

Find out who is correct and who is wrong.

1. Jamil says: $\frac{4}{6}$ of 12 is the same as $\frac{2}{3}$ of 12.

2. Katy says: $\frac{2}{5}$ of 20 is more than $\frac{3}{4}$ of 16.

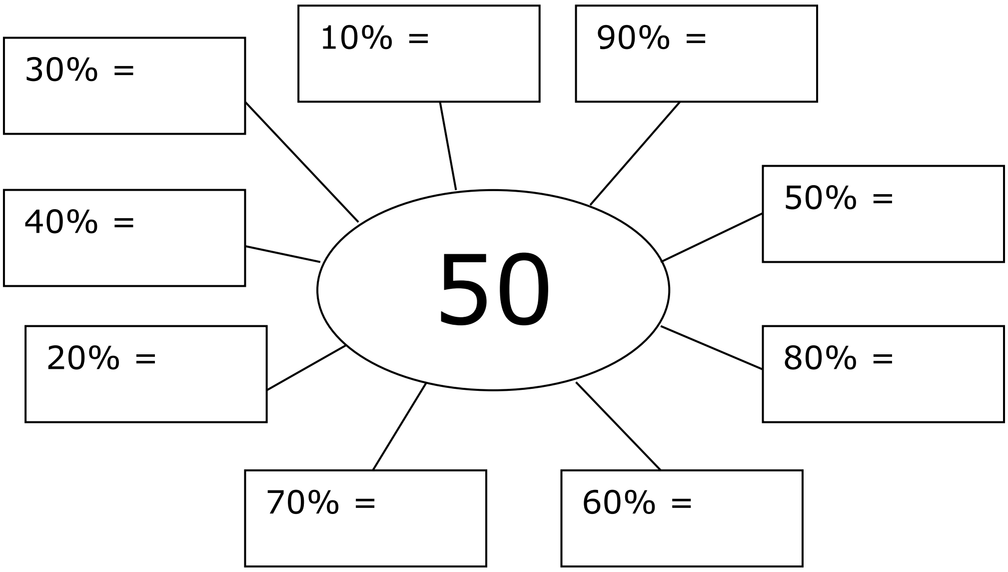
3. Rael says: $\frac{3}{10}$ of 20 is less than $\frac{1}{3}$ of 18.

4. Sara says: $\frac{4}{5}$ of 15 is more than $\frac{5}{8}$ of 16.

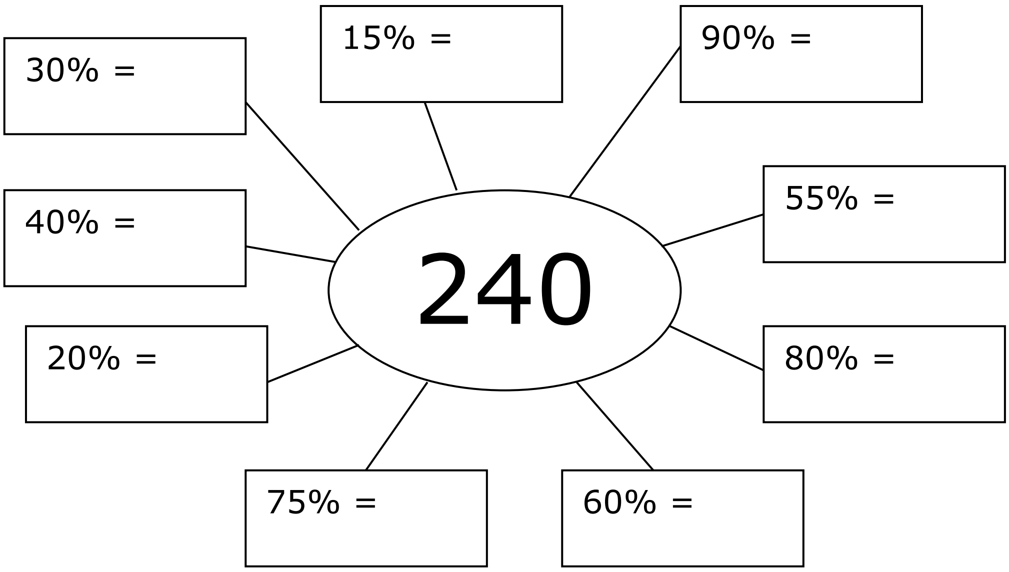
5. Write correct statements to replace those which are wrong.

Simple percentage problems

1. Find these percentages of 50. Which ones are the easiest to find first?



2. What is 10% of 240? 5%? 50%? 25%? Use these to complete this.
You may need a calculator for some of them.



Decimal problems using a calculator

1. I have 3 pencils. They are 12.5 cm, 13.4 cm and 6.8 cm long.
If I lay them end to end, will they be longer or shorter than my 30 cm ruler?
How much longer or shorter?

2. A truck is 2.9 m high.
Will it be able to go under a bridge which is 3.3 m high?
How much space will there be between the top of the truck and the bottom of the bridge?

3. It is 3.4 km to the railway station if I walk directly from my house.
My friend lives 1.8 km from my house and 2.9 km from the station.
How much further will I have to walk if I go to my friend's house before going to the station?

4. I am making some bookshelves to fit into an alcove which is 1.4 m wide.
I have 2 pieces of suitable wood. One is 1.6 m long and the other is 2.9 m long.
How many shelves can I make?
How should I cut the wood?

Access Unit 12: Measures 1

Objectives

a) **Estimate, measure and compare lengths, using standard units; suggest suitable units and equipment for such measurements.**

b) **Read a simple scale to the nearest labelled division, including using a ruler to draw and measure lines to the nearest centimetre.**

c) **Find areas by counting squares.**

Include shapes with $\frac{1}{2}$ squares.

d) **Convert between digital and analogue times using the 12-hour clock (i.e. tell the time).**

e) **Use units of time and know the relationships between them (second, minute, hour, day, week, month, year).**

f) **Find the perimeter of a shape by adding lengths of sides.**

Keywords

estimate, measure, compare, length, centimetre, metre, scale, area, digital, analogue, second, minute, hour, day, week, month, year, perimeter, side

Common misconceptions

Confuses area and perimeter.
Support by talking about 'the area' of a football pitch, or 'the area' where you live and perimeter fences which go round an area.

Does not understand 'to' times, e.g. 5 to 6.
Use an analogue clock with movable hands to illustrate analogue time.

Writes 5 past times without zero, e.g. 5 past 4 as 4:5.

Measures from point other than zero on ruler.

Measures 1 check-up

1. Measure this line to the nearest cm.

2. Draw a line 12 cm long.

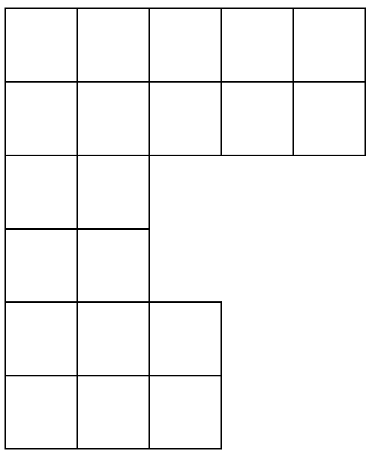
3. What unit would you use to measure these?
The length of a football pitch

The distance to the nearest railway station

The length of a pencil

The length of an ant’s leg

4. What is the area of this shape? (Assume squares are 1 cm^2.)
What is its perimeter?



5. Write these times as they would appear on a digital watch.

6. How many hours are in one day?

7. How many days are there in February in a leap year?

8. How often is a leap year?

9. How do you know if you are in a leap year?

10. How many months are in one year?

11. How many minutes are in one hour?

12. How many seconds are in half a minute?

Measures 1

Opportunities for reasoning/problem solving – teacher notes

Investigation of area and perimeter of rectangles

The student will need centimetre squared paper for these. The first of these investigations is very directed, while the second requires the student to take their own decisions about what to draw.

Answers

1. Rectangles with an area of 12 cm^2 are 3×4, 2×6 and 1×12. These have perimeters of 14, 16 and 26 cm respectively. So only B is correct.

2. If a student is unsure what to do for the second part, ask, 'What squares could you draw?' Any square (except 1×1) can be used to explore the statements.

In the simplest case: a square 2×2 (area 4 cm^2, perimeter 8 cm) with oblongs 1×4 (area 4 cm^2) and 1×3 (perimeter 8 cm) show that neither D nor E is correct.

Time problems using a calculator

The student may need support in understanding which operation they need to key into the calculator. Most of these are simply multiplication, though Q1 and Q7 also involve addition. The student should be encouraged to read the numbers to give practice in translating the numbers into the spoken word.

Answers

1. 1826
2. 1440
3. 1800
4. 1200
5. 672
6. 950 (assuming a 25-lesson week)
7. This will depend on the age of the student and the time of the year.

Investigation of area and perimeter of rectangles

You are going to look at the area and perimeter of some rectangles to investigate what happens.

1. Here are some people's predictions about the area and perimeter of rectangles.

I think that if rectangles have the same area, they will have the same perimeter.

A

I think a short fat rectangle will have the same area as a long thin one, but the long thin one will have a longer perimeter.

B

I don't think rectangles can have the same area if they have different sides.

C

What do you think?

Draw rectangles with an area of 12 square centimetres.
For each one, work out the perimeter and write it by the rectangle.

Is A correct? What about B and C? What do you think now?

2. Here are some more predictions.

A square can't have the same perimeter as an oblong.

D

A square can't have the same area as an oblong.

E

Draw some squares and oblongs to check whether D or E is correct.
(An oblong is a rectangle which isn't a square.)

Time problems using a calculator

Use your calculator to work these out.

1. How many days are there in 5 years, if only one of these years is a leap year?

2. How many minutes are in one day?

3. How many seconds are in half an hour?

4. How many months are in a century?

5. How many hours are in February (not in a leap year)?

6. How many lessons do you have in a week? How many in a year? (Assume you are at school for 38 weeks of the year.)

7. How many days have you been alive? (You may need a calendar to help you work this out.)

Access Unit 13: Measures 2

Objectives

a) Know and use the relationships between familiar units of length, i.e. mm, cm, m, and km.

b) Find the area of a rectangle by multiplying length × width.

c) Classify acute and obtuse angles.

Set of angles

See the page of angles provided for practice with this (after Common misconceptions). For best results, laminate them before cutting out.

Ask the student what kind of angles they know and develop the headings: acute, obtuse, right. Ask them to sort the angles under the headings. For those angles that are close to a right angle, show them how to overlay them with the corner of a piece of paper to establish whether they are exactly a right angle.

When they move on to measuring angles, they may need to be reminded of this exercise.

d) Use a protractor to measure acute and obtuse angles to the nearest degree.

e) Use a protractor to measure angles in a triangle to show practically that the angle sum is 180°.

f) Use a ruler to draw lines to the nearest millimetre.

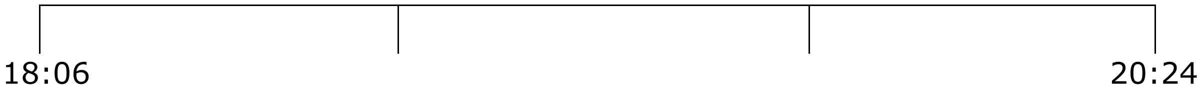
g) Use a pair of compasses to draw a circle.

h) Convert between 12-hour and 24-hour clock times.

i) Calculate simple time intervals using a time line.

e.g. A film starts at 18:06 and finishes at 20:24. How long is the film?

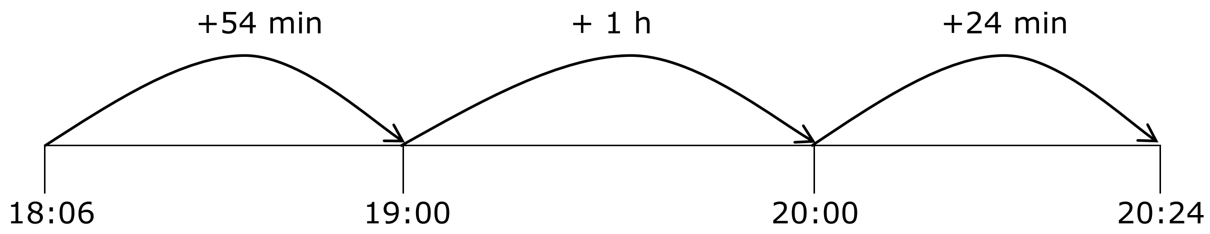
Start with a time line like this:



Ask what the signposts might be between these times. If they are not clear, ask what the next hour would be.

This should lead to a labelled time line:

Then establish what the jumps are:



So the answer is 1 h and 78 min which is 2 hours and 18 minutes.

Emphasise that care must be taken with writing the units and then putting them together.

j) Find volume by counting cubes.

k) Read scales between labelled divisions.

Keywords

estimate, measure, compare, length, centimetre, metre, scale, area, digital, analogue, second, minute, hour, day, week, month, year, perimeter, side, **millimetre**, **kilometre**, **width**, **angle**, **acute**, **obtuse**, **protractor**, **degree**, **triangle**, **angle sum**, **circle**, **pair of compasses**, **12-hour**, **24-hour**, **convert**, **volume**

Common misconceptions

Confuses area and perimeter.
Support by talking about 'the area' of a football pitch, or 'the area' where you live and perimeter fences which go round an area.

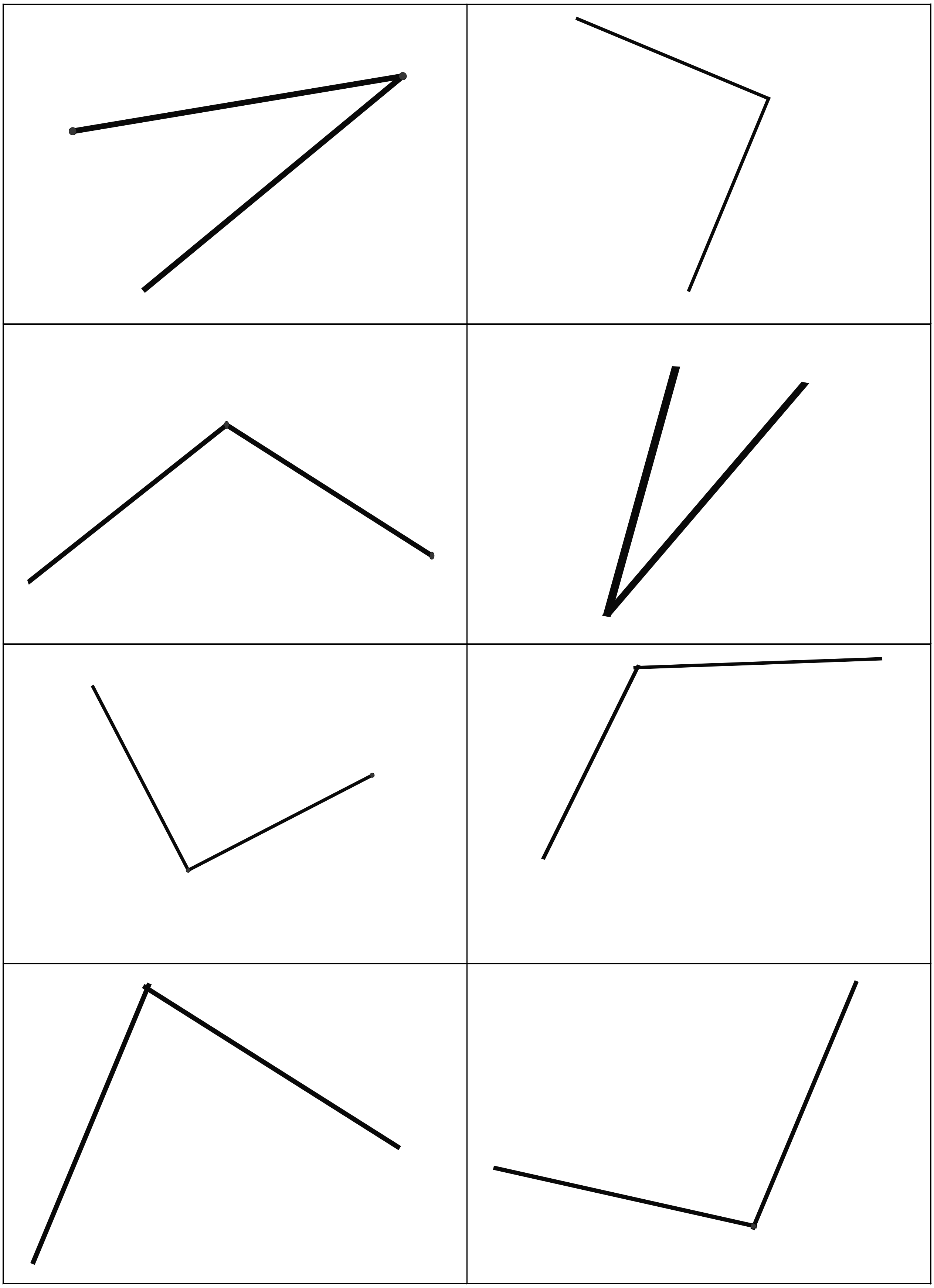
Does not understand 'to' times, e.g. 5 to 6.
Use an analogue clock with movable hands to illustrate analogue time.

Writes 5 past times without zero, e.g. 5 past 4 as 4:5.

Measures from point other than zero on ruler.

Has difficulty in using protractor, even though can say an angle is acute or obtuse, or uses the wrong scale.
Help by showing that one side of the angle needs to start from zero: this determines which scale to use, then check, 'Is this angle acute/obtuse?'

Misreads scales.
Emphasise the importance of determining what each division represents.



Measures 2 check-up

1. Measure this line to the nearest mm.

2. Draw a line 48 mm long.

3. How many:

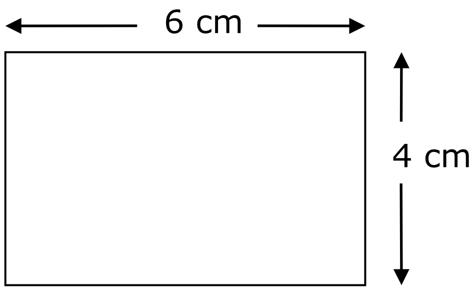
mm in 1 cm

cm in 1 metre

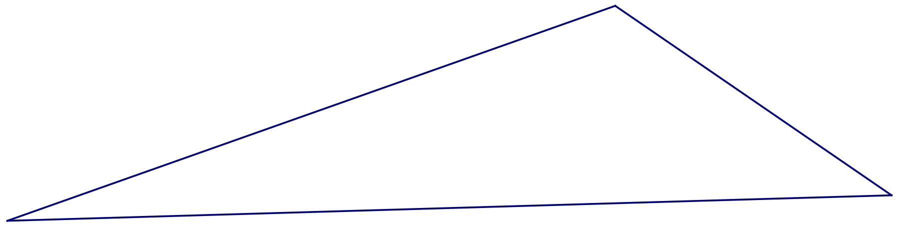
metres in 1 km

mm in 1 metre

4. Find the area of this rectangle.



5. Measure the angles in this triangle to the nearest degree.



What kind of angles are they? Add them together.

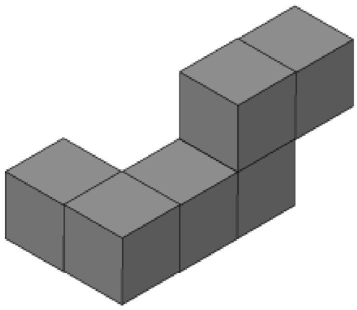
6. Change these times to 24-hour clock times:
Ten to 9 in the morning
Quarter past 2 in the afternoon

How many hours and minutes are there between these times?

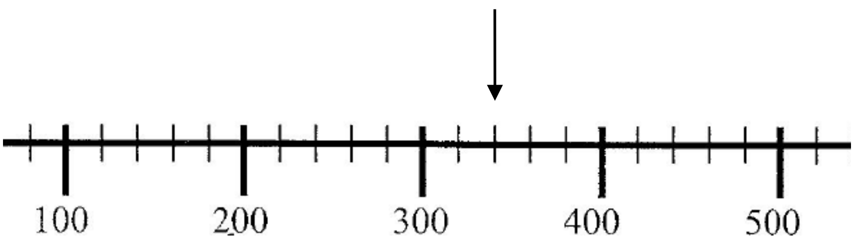
7. Change these 24-hour clock times to am or pm times:
09:50
17:10

How many hours and minutes are there between these times?

8. What is the volume of this shape?



9. Write down the number shown by the arrow.



Measures 2

Opportunities for reasoning/problem solving – teacher notes

Triangle angle sum puzzles

These are some simple puzzles which reinforce that the angles of a triangle add to 180° in the simple context of right-angled triangles. Some students may realise that the other two angles need to add to 90°, thus making the problems simpler.

Answers

1. 60°

2. 45°

3. 30°

4. 35°

5. 70°

Cuboid puzzles

The student will need at least 24 linking cubes. If they have difficulty starting, ask them what factors of 24 they know, and suggest they start from there. They may want to draw the cuboids and isometric paper enables them to draw them in 3D.

Answers

There are 6: $1 \times 1 \times 24$, $1 \times 2 \times 12$, $1 \times 3 \times 8$, $1 \times 4 \times 6$, $2 \times 2 \times 6$, $2 \times 3 \times 4$

1. 6

2. Rectangles, some of which are squares

3. 8

4. 12

5. Any of the cuboids with a dimension of 2 will also have a face with an area of 24

6. 24 cubes

7. Cube

8. 27

9. Squares

Circle designs

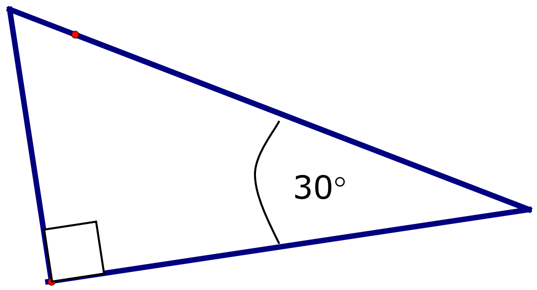
The purpose of this exercise is to give practice with using a pair of compasses, measuring and keeping the compasses fixed while drawing. Encourage the student to practise until they get a pleasing result.

Triangle angle sum puzzles

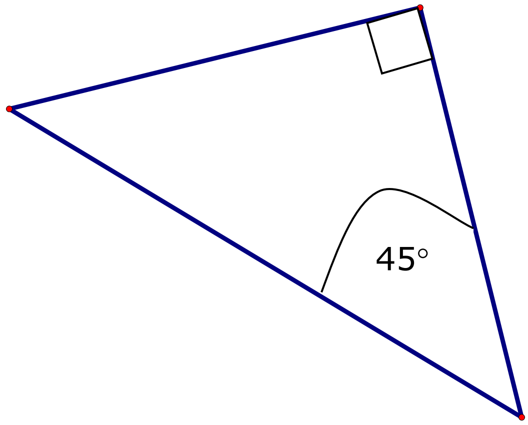
These triangles all have a right angle. Can you work out the missing angles?

They are not drawn to scale, so you can't find the answers by measuring – you have to work them out.

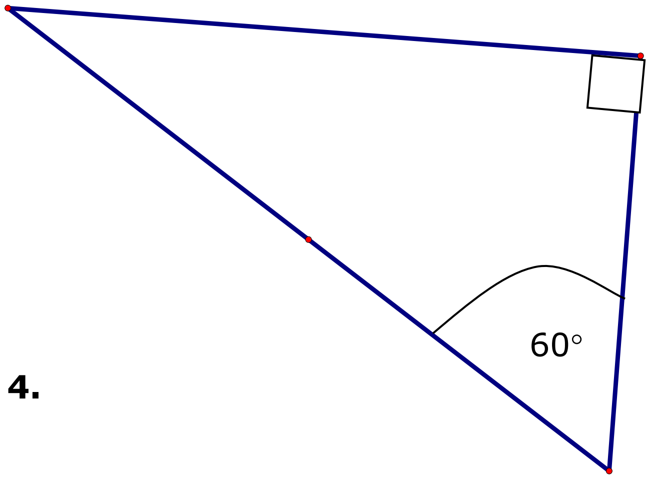
1.



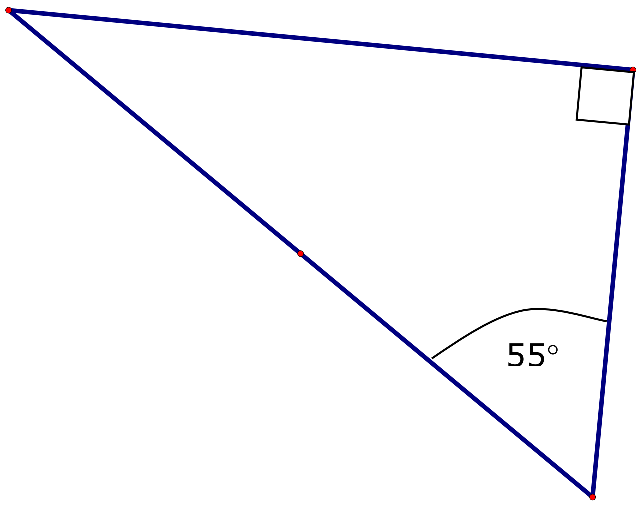
2.



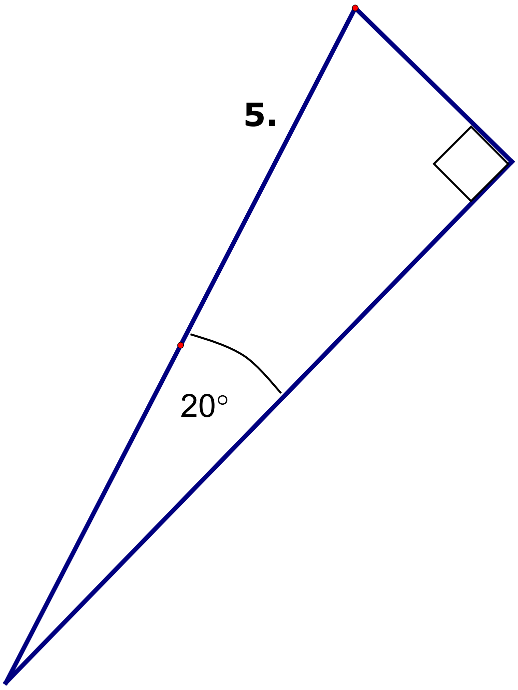
3.



4.



5.



Cuboid puzzles

How many different cuboids can you make with 24 linking cubes?

Think about how you will record the cuboids on paper. You might draw them on isometric paper, or you might find a way of writing down their dimensions so that each is clear.

For each cuboid:

1. How many faces does it have?

2. What shape are the faces?

3. How many corners (vertices) does it have?

4. How many edges does it have?

5. Which of the cuboids have a face with the largest area?

6. What is the volume in cubes of each cuboid?

7. I make a different cuboid with the dimensions $3 \times 3 \times 3$. This is a special cuboid – what is its name?

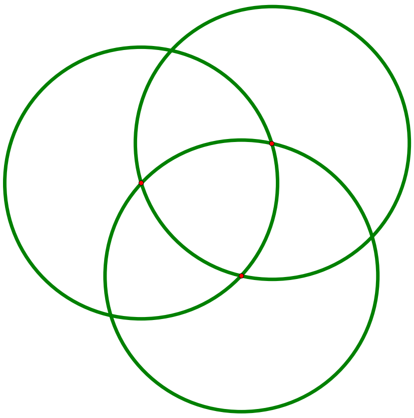
8. How many cubes do I need to make it?

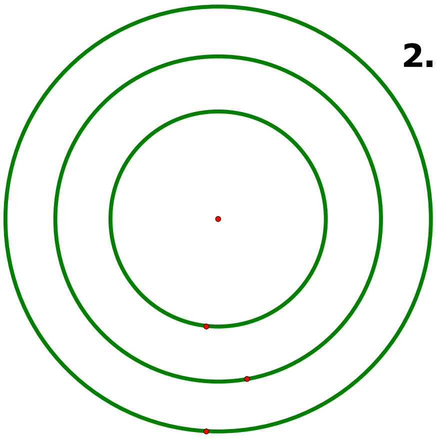
9. What shape are its faces?

Circle designs

Use a pair of compasses to make these patterns.

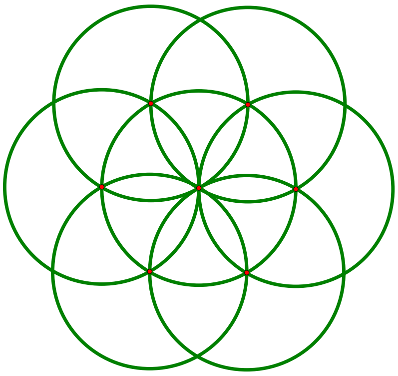
1. Have your compasses set at 4 cm for this pattern.





2. Set your compasses at 3 cm, then 4 cm, then 5 cm for this pattern.

3. Set your compasses at 3.5 cm for this pattern.



4. Make up your own circle patterns.

Access Unit 14: Shape 1

Objectives

a) **Use the mathematical names for common 2D shapes (triangle, square, rectangle, hexagon, circle) and 3D shapes (cube, cuboid, pyramid, sphere); sort shapes and describe some of their features (using vocabulary: side, angle, face, edge).**
Classify triangles; know that a square is a special rectangle.
Understand the difference between 2D and 3D shapes.

b) **Identify right angles.**

Keywords

2D, 3D, isosceles, equilateral, scalene, right-angled triangle, square, rectangle, circle, right angle, hexagon, cube, cuboid, sphere, pyramid, face, edge

Common misconceptions

Cannot remember names of triangles.

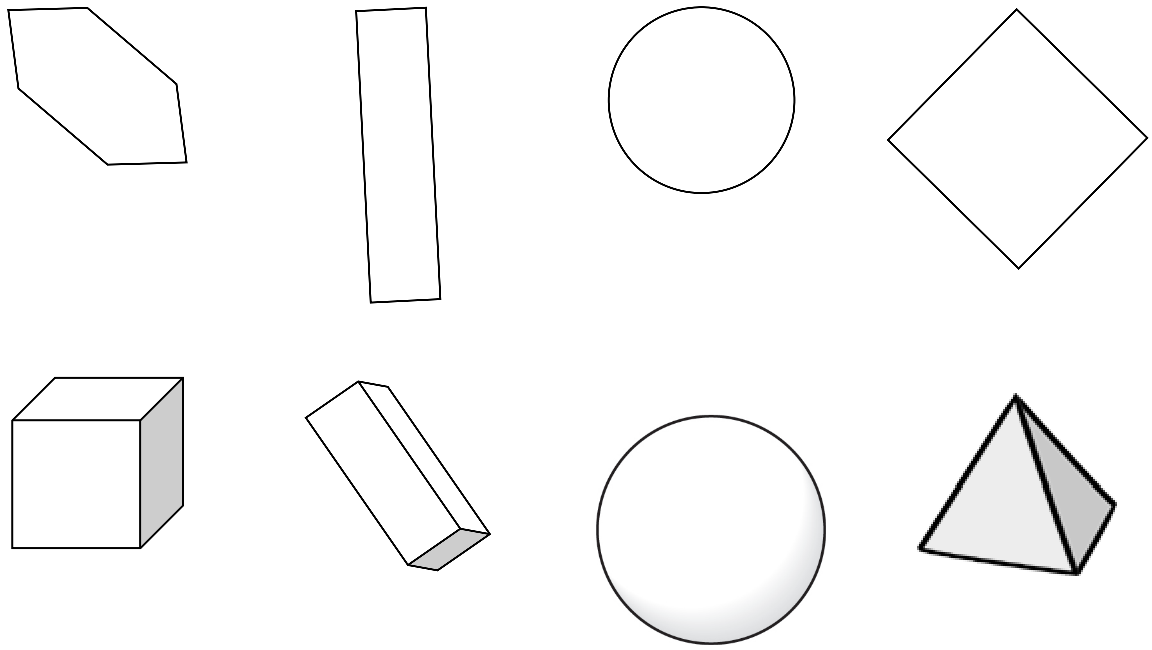
Knows the properties of the sides of different triangles, but not the angle properties.

Cannot identify right angles when they are not square to a page.

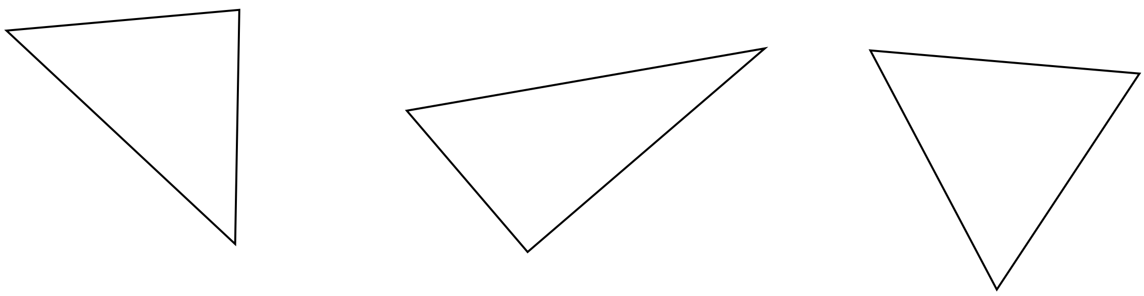
Does not recognises shapes when presented at an unusual angle, e.g. calls a tilted square a diamond, thinks that a rectangle must always have its long sides horizontal or that one side of a triangle must be horizontal.

Shape 1 check-up

1. Name these shapes.



2. Label these triangles.



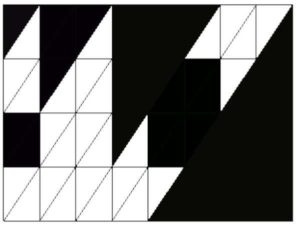
3. What characteristics do all rectangles share?

Shape 1

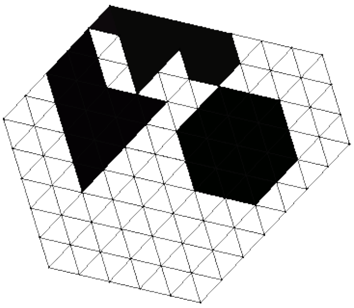
Opportunities for reasoning/problem solving – teacher notes

Finding shapes on a grid

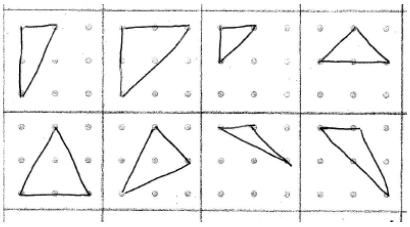
1. Four different sized right-angled triangles can be made, as well as hexagons and rectangles of different sizes. Trapezia and parallelograms can also be made, but the student may not have met these yet; they are covered in Unit 15 Shape 2.



2. Equilateral triangles and hexagons can be made, as well as parallelograms, including rhombuses and trapezia. The student may also see the 3D possibilities of the hexagon being seen as a cube, and begin to draw cuboids.



Making triangles on a pinboard

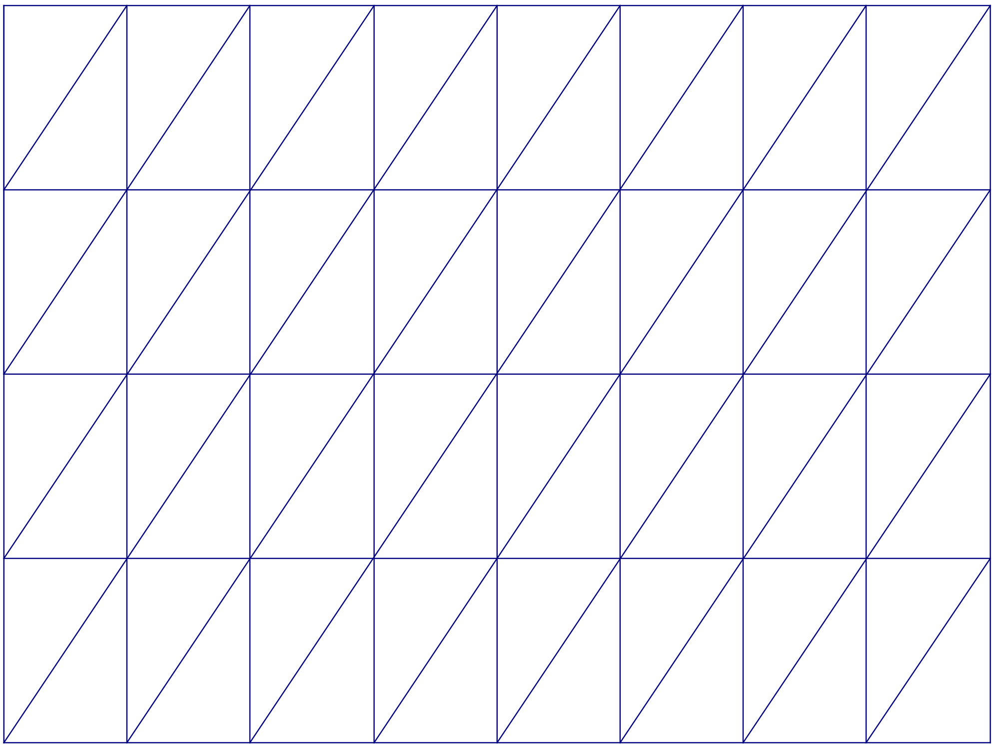


The student may find the last three difficult to find. They could be encouraged to start with one side and find all the possibilities for the 3rd vertex. They may also find the same triangles in different orientations – tracing paper may help to convince them they are the same.

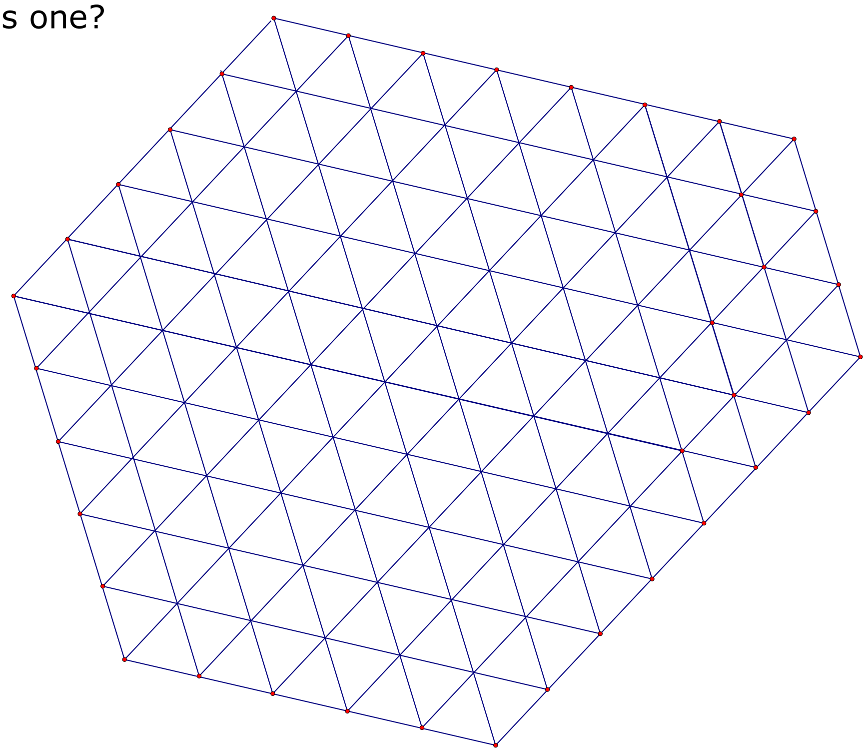
1. 8 triangles can be made using 9 dots.
2. 4 are right-angled triangles.
3. 3 of the 4 right-angled triangles are isosceles, as are 2 others.
4. The other right-angled triangle is scalene, as are 2 others.
5. 2 of the scalene triangles have obtuse angles.
6. No.

Finding shapes on a grid

1. What shapes can you find on this grid?



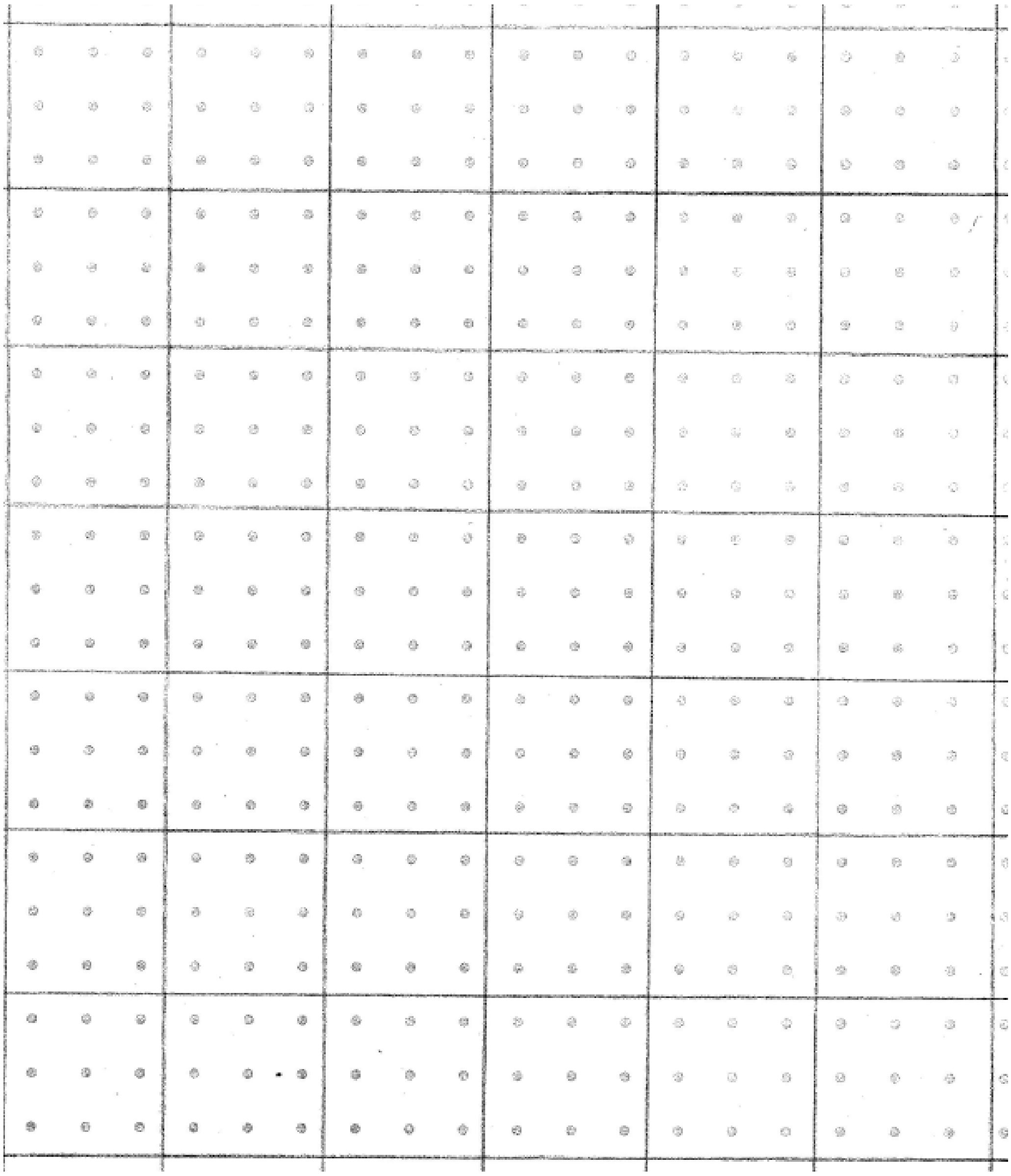
2. What about this one?



Making triangles on a pinboard

Use the pinboard to investigate making triangles.

1. What triangles can you make using 9 dots?
2. How many are right-angled?
3. How many are isosceles?
4. Are any of the triangles scalene?
5. How many have obtuse angles?
6. Can you make an equilateral triangle on a square grid?



Access Unit 15: Shape 2

Objectives

a) **Classify polygons, using criteria such as number of right angles, whether or not they are regular, symmetry properties.**
   Classify quadrilaterals.

b) **Recognise parallel and perpendicular lines, and properties of rectangles.**

Keywords

2D, 3D, isosceles, equilateral, scalene, right-angled triangle, square, rectangle, circle, right angle, hexagon, cube, cuboid, sphere, pyramid, face, edge, **regular**, **irregular**, **quadrilateral**, **parallel**, **perpendicular**, **parallelogram**, **rhombus**, **kite**, **trapezium**, **property**, **symmetry**, **line of symmetry**, **reflection symmetry**, **rotational symmetry**

Common misconceptions

Cannot remember names of triangles.

Knows the properties of the sides of different triangles, but not the angle properties.

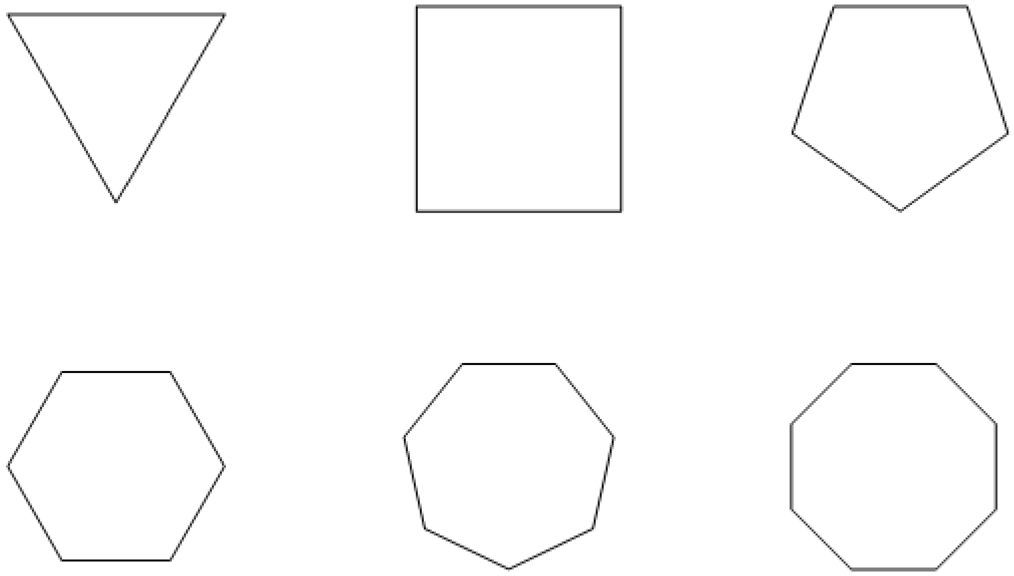
Cannot identify right angles when they are not square to a page.

Does not recognises shapes when presented at an unusual angle, e.g. calls a tilted square a diamond, thinks that a rectangle must always have its long sides horizontal or that one side of a triangle must be horizontal.

Calls a rhombus a diamond.

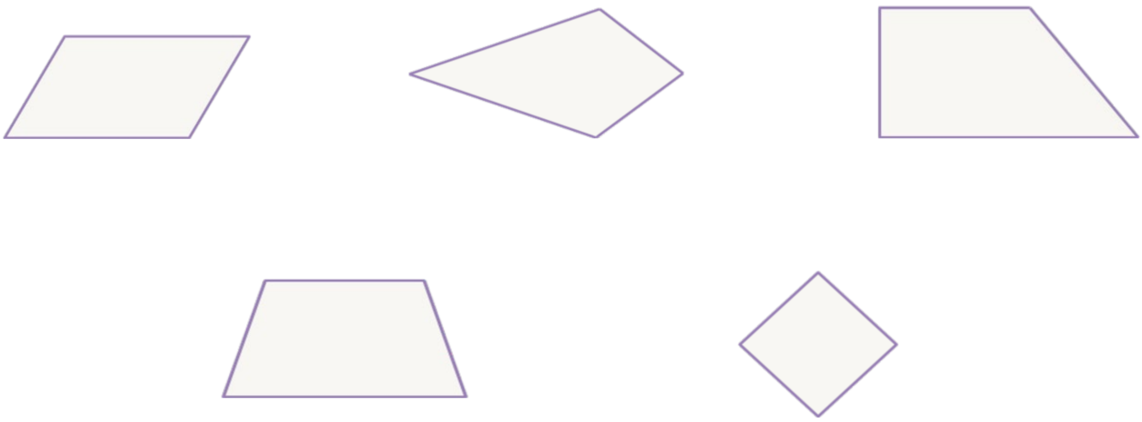
Shape 2 check-up

1. Name these regular shapes.



Mark lines that are parallel where appropriate.

2. Name these quadrilaterals.



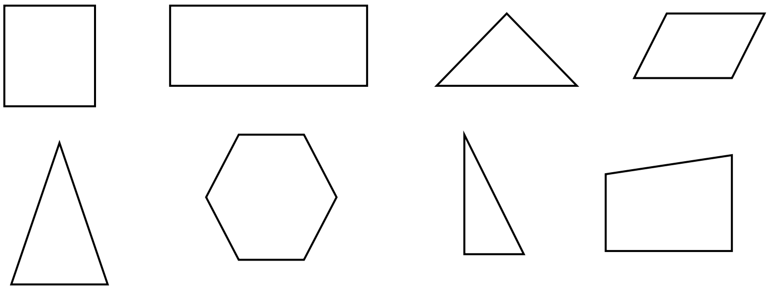
Shape 2

Opportunities for reasoning/problem solving – teacher notes

Carroll diagrams for sorting shapes

Students sometimes have difficulty because they have to consider two properties at the same time. It can help if they start with one property, sort the shapes according to that property and then consider the second property.

If the student finds this very difficult you might consider starting with something simpler, such as sorting these shapes into the table below.



No right angles	1 right angle	More than 1 right angle

Answers

1.

	At least 1 right angle	No right angles
At least 2 equal sides	square, rectangle, isosceles right-angled triangle, irregular hexagon (with 2 right angles), irregular pentagon (with 2 right angles	isosceles triangle, equilateral triangle, parallelogram, rhombus, regular hexagon, regular pentagon, isosceles trapezium
No equal sides	trapezium (with 2 right angles), scalene right-angled triangle	student's own drawing – e.g. irregular quadrilateral with no right angles or equal sides OR scalene triangle with no right angle OR non-isosceles trapezium with no right angle

2.

	At least 2 parallel sides	No parallel sides
At least 2 perpendicular sides	square, rectangle, trapezium (with 2 right angles), irregular hexagon (with 2 right angles)	kite (with 1 right angle), kite (with 2 right angles), irregular pentagon (with 2 right angles)
No perpendicular sides	regular hexagon, rhombus, isosceles trapezium	regular pentagon, kite

Other properties which could be used in follow-up exercises could be:

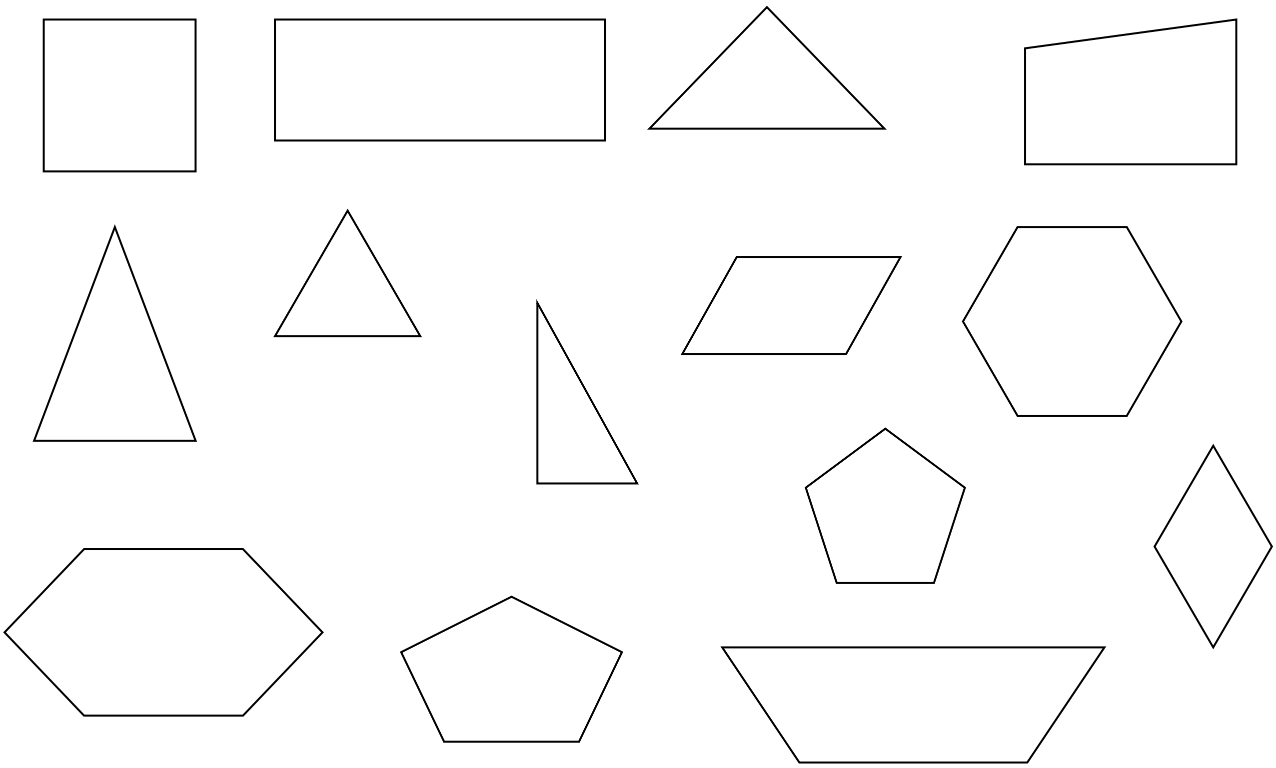
- rotational symmetry/no rotational symmetry
- reflective symmetry/no reflective symmetry
- rotational symmetry order 1, 2, or more than 2.

Venn diagrams can also be used for sorting shapes, e.g. with these headings:
At least 1 right angle/At least 2 equal sides.

Carroll diagrams for sorting shapes

1. Sort the shapes below onto this diagram.

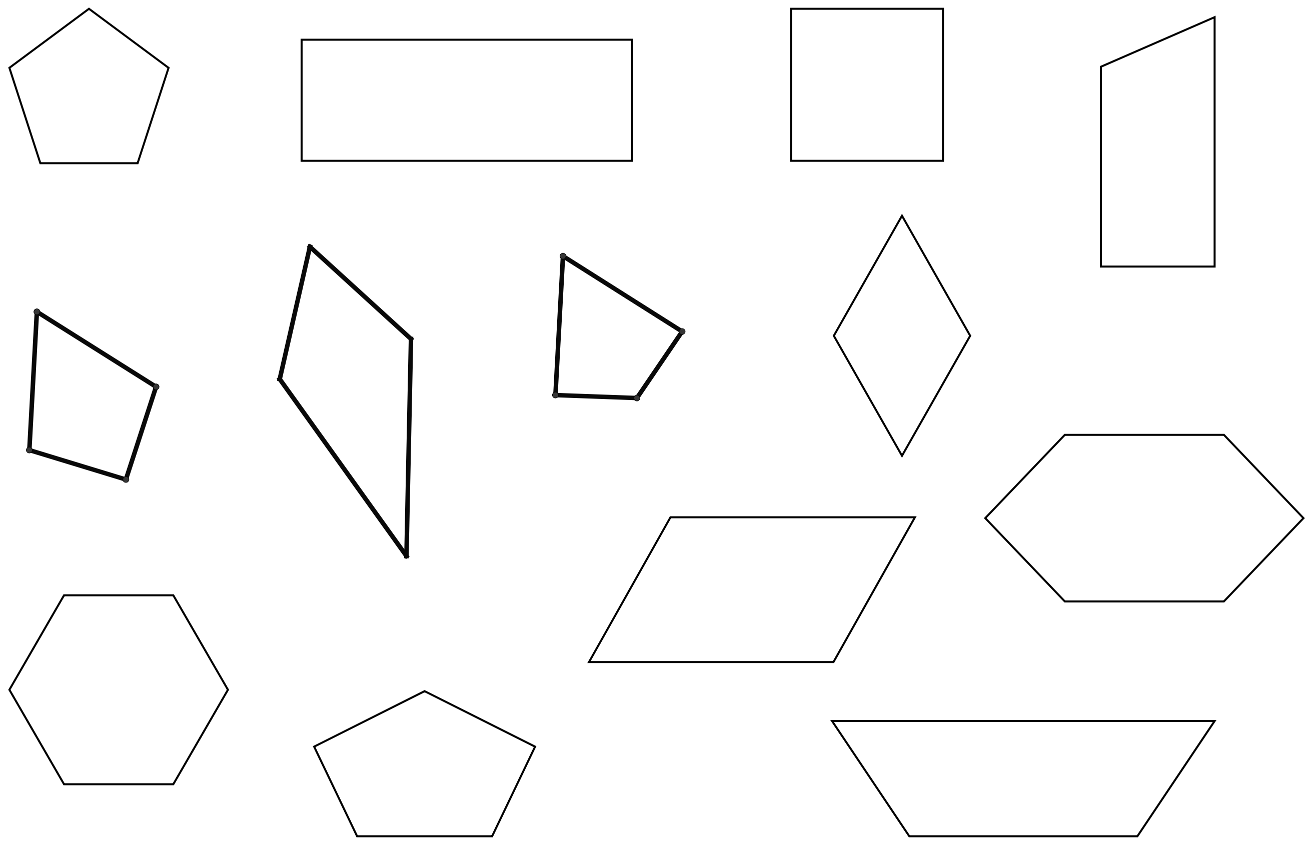
	At least 1 right angle	**No right angles**
At least 2 equal sides		
No equal sides		



Can you draw a shape for the empty cell?

2. Sort the quadrilaterals below and other polygons onto this diagram.

	At least 2 parallel sides	**No parallel sides**
At least 2 perpendicular sides		
No perpendicular sides		



Access Unit 16: Transformations 1

Objectives

a) Use mathematical vocabulary to describe position, direction and movement (including clockwise and anti-clockwise).

Keywords

turn, clockwise, anti-clockwise, direction, left, right, forwards, backwards, sideways, whole turn, half turn, quarter turn

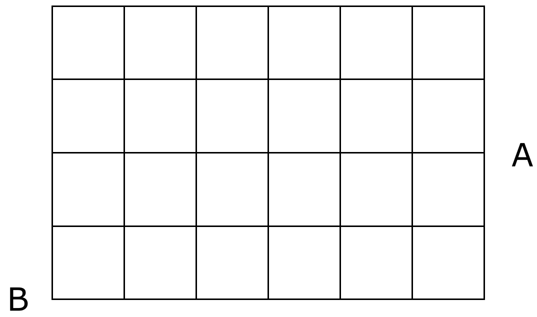
Common misconceptions

Confuses left and right, clockwise and anti-clockwise.

Has difficulty in directing someone who is facing them: transposes left and right.

Transformations 1 check-up

1. Direct a robot from A to B along lines in the grid.



The robot understands:

Take the left path (L)
Take the right path (R)
Forward (F) a given number of squares

Direct the robot back, from B to A, along a different path.

2. Direct a blindfolded student around an obstacle course.

Transformations 1

Opportunities for reasoning/problem solving – teacher notes

Clock investigation

Some students will be supported with this activity by having a clock whose hands they can actually move.

Answers

1. 1

2. 1

3. 8

4. 11

5. 12

6. They meet at 4 if the hand at 10 makes a $\frac{1}{2}$ turn clockwise or anti-clockwise and the hand at 7 makes a $\frac{1}{4}$ turn anticlockwise. Or, they meet at 1 if the hand at 10 makes a $\frac{1}{4}$ turn clockwise and the hand at 7 makes a $\frac{1}{2}$ turn clockwise or anti-clockwise.

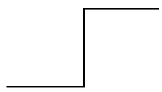
7. The hand at 3 makes a $\frac{1}{4}$ turn anti-clockwise, the hand at 6 makes a $\frac{1}{2}$ turn clockwise or anti-clockwise, and the hand at 9 makes a $\frac{1}{4}$ turn clockwise.

How many different ways?

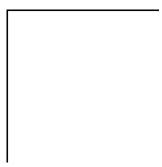
It may help the student to draw these different ways out on squared paper before writing the instructions.

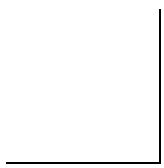
Answers

1. 3 ways: L F1 R F2 R F2 L F1 R F1 L F1 R F1

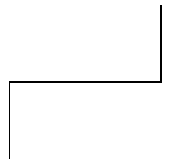


2. 6 ways: L F2 R F2 R F2 L F2

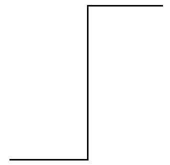




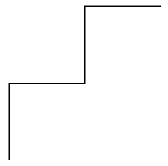
L F1 R F2 L F1



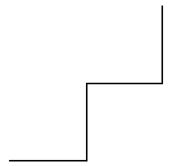
R F1 L F2 R F1



L F1 R F1 L F1 R F1

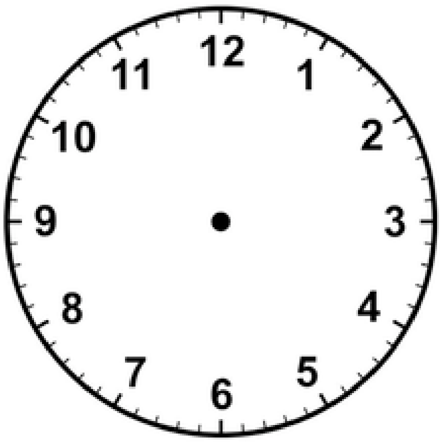


R F1 L F1 R F1 L F1



3. 7 blocks

Clock investigation



1. A clock hand starts at 7 and takes half a turn clockwise.
   Where is it now?

2. A hand starts at 4 and takes a quarter turn anti-clockwise.
   Where is it now?

3. A hand starts at 11 and takes a quarter turn clockwise and then a half turn anti-clockwise.
   Where is it now?

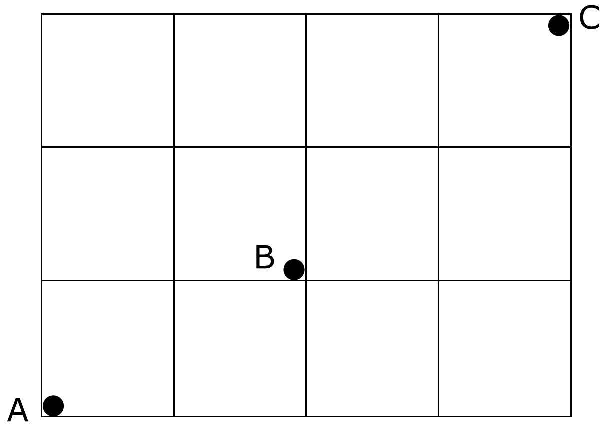
4. A hand takes half a turn clockwise and finishes at 5.
   Where did it start?

5. A hand finishes at 9 after a quarter turn anti-clockwise.
   Where did it start?

6. One hand starts at 10 and another at 7.
   What turns do they have to make to meet each other? They can only take $\frac{1}{2}$ or $\frac{1}{4}$ turns. Where will they meet?
   Is there another way they can turn and meet?

7. There are hands at 3, 6 and 9.
   What turns do they have to make to meet at 12?

How many different ways?



A robot is employed in a field where it starts from A, goes to B to pick up animal food and then goes to C to drop the food for the animals. It always takes the shortest path possible.

It only understands these commands:

Take the left path (L)
Take the right path (R)
Go forward 1 block (F1)
Go forward 2 blocks (F2)

1. How many ways are there for it to go from A to B?
Write instructions for the robot using the commands above.

2. How many ways are there for it to get from B to C?
Write instructions using the same commands.

3. Whatever route the robot takes from A to C, it always goes the same number of blocks. How many blocks?

Access Unit 17: Transformations 2

Objectives

a) Identify lines of symmetry in simple shapes and recognise shapes with no lines of symmetry.

Keywords

line of symmetry, symmetrical

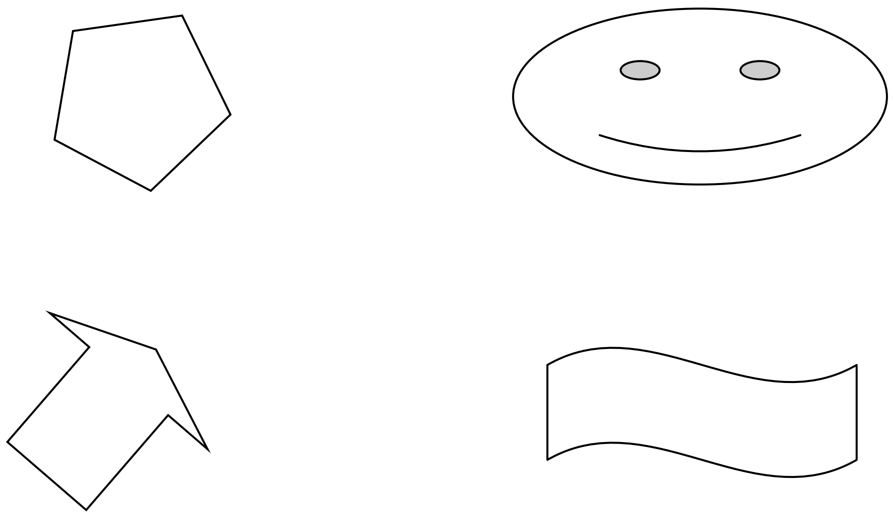
Common misconceptions

Has difficulty in ‘seeing’ symmetry.
Use a mirror to demonstrate.

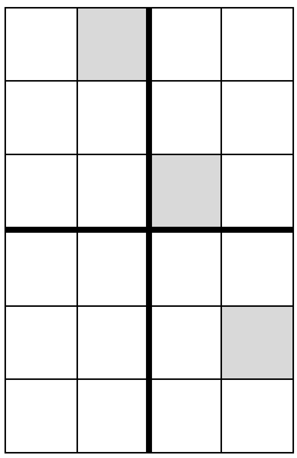
When using a mirror, thinks that everything is symmetrical because the image in the mirror matches the object.

Transformations 2 check-up

1. Which of these shapes have line symmetry?
Draw the lines of symmetry.



2. Complete this pattern so that it has 2 lines of symmetry.



Transformations 2

Opportunities for reasoning/problem solving – teacher notes

Investigation of the symmetry of regular shapes

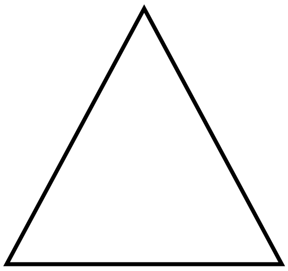
For regular shapes the number of sides is the same as the number of lines of symmetry. This might seem obvious, but some students will have difficulty in finding them all. Having larger cut-out shapes which they can fold may help. Some may find a mirror helpful.

A circle has an infinite number of lines of symmetry.

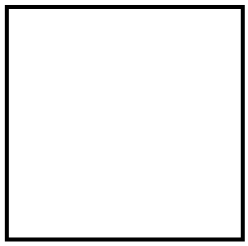
Investigation of the symmetry of regular shapes

Here are some regular shapes.
How many lines of symmetry can you find on each one?

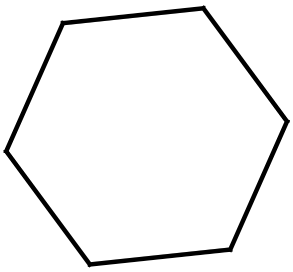
An equilateral triangle



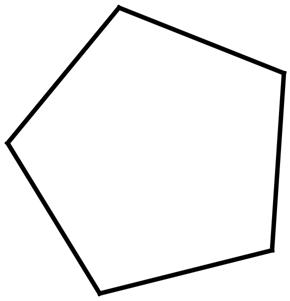
A square



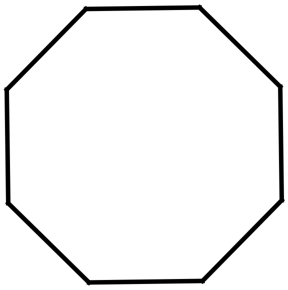
A regular hexagon



A regular pentagon

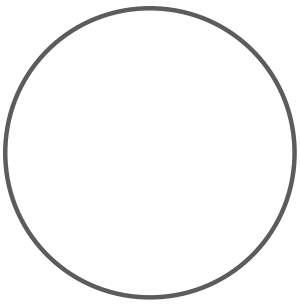


A regular octagon



What do you notice?

What about a circle?



Access Unit 18: Transformations 3

Objectives

a) Identify order of rotational symmetry in simple shapes and complete patterns with a given order of rotational symmetry.

Keywords

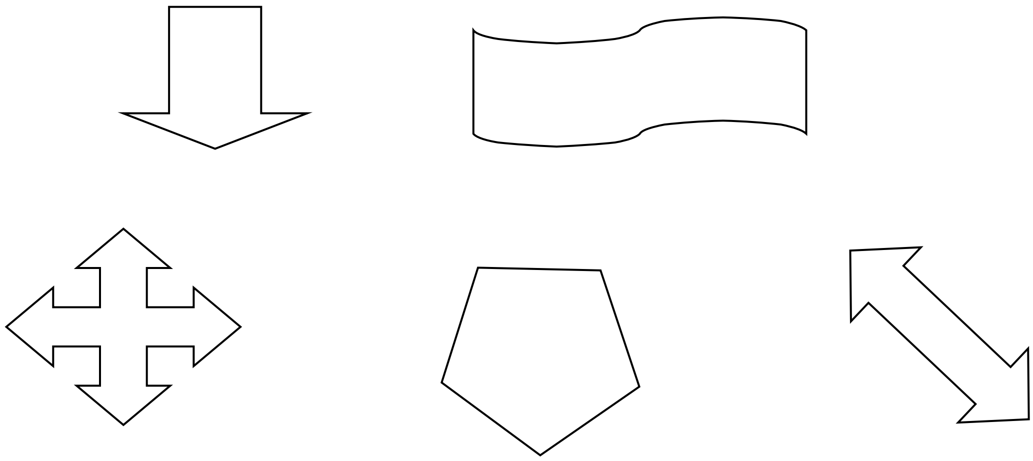
turn, clockwise, anti-clockwise, direction, left, right, whole turn, half turn, quarter turn, **rotation**, **order of rotational symmetry**, **right angle**, **degrees**

Common misconceptions

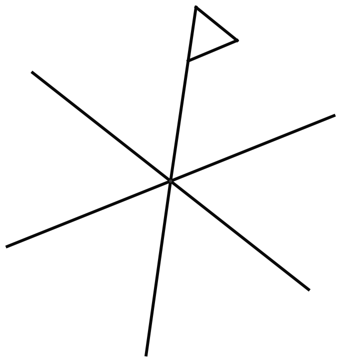
Finds it difficult to 'see' rotational symmetry.
Support with tracing paper to demonstrate matching after turn.

Transformations 3 check-up

1. What order of rotational symmetry do these shapes have?



2. Complete this pattern so that it has rotational symmetry order 3.



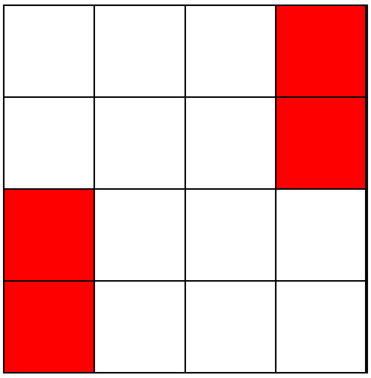
Transformations 3

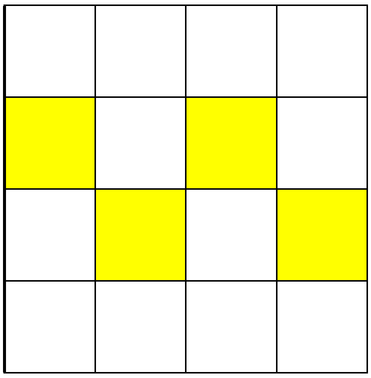
Opportunities for reasoning/problem solving – teacher notes

Combinations of reflection and rotational symmetry

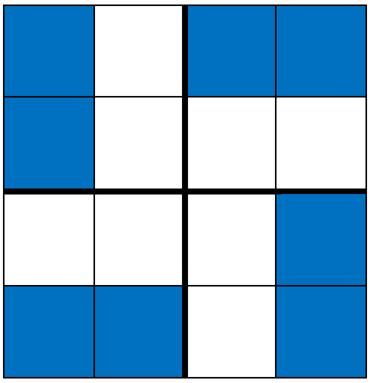
Answers

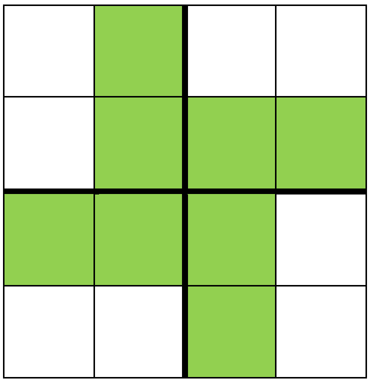
1. Order 2



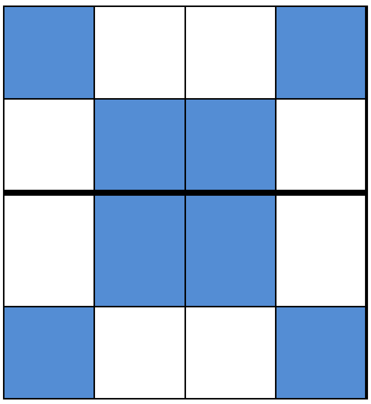


2. Order 4

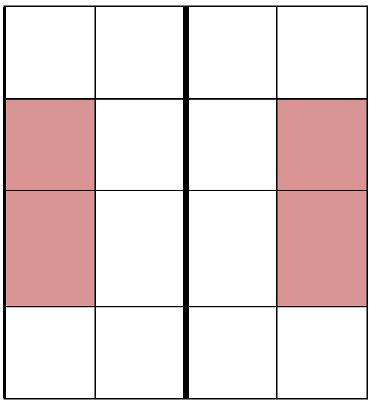




3. Note: In both cases other squares may be shaded but the whole pattern must have the required symmetry.



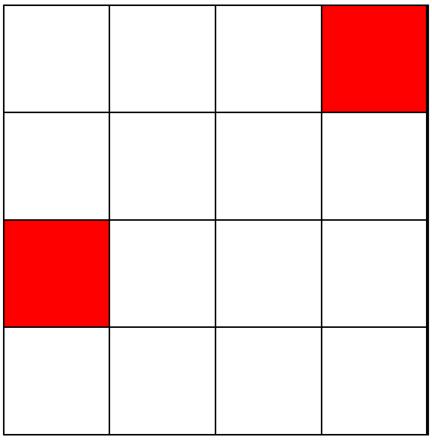
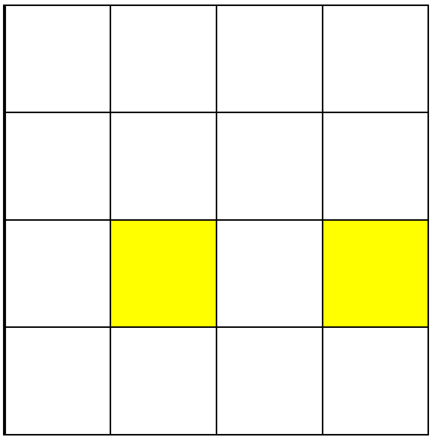
Rotational order 4



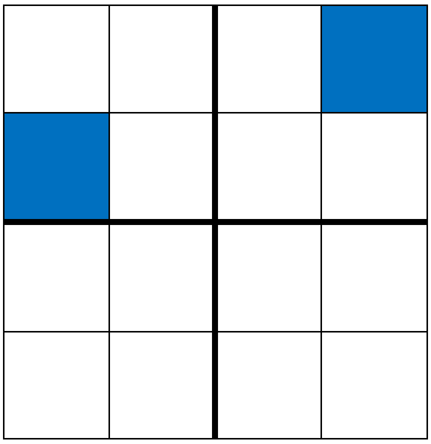
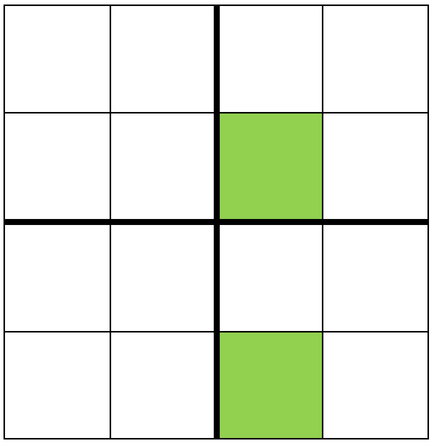
Rotational order 2

Combinations of reflection and rotational symmetry

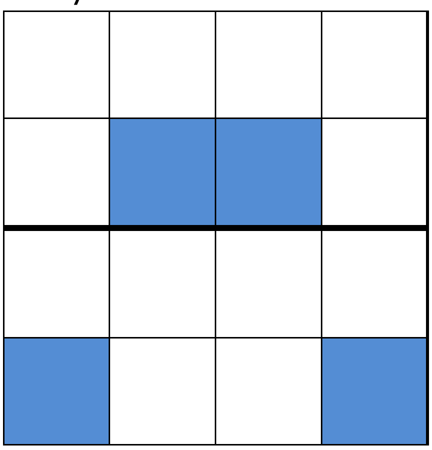
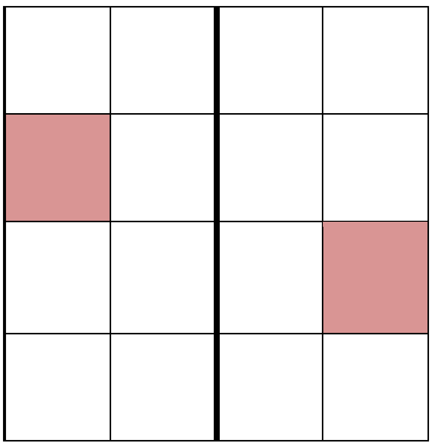
1. Complete these patterns so that they have rotational symmetry order 2.

2. Complete these patterns so that they have rotational symmetry order 4.

3. Complete these patterns so that the dark line is a line of reflection symmetry.

What order of rotational symmetry do they have?

Access Unit 19: Coordinates 1

Objectives

a) Read and plot coordinates in the first quadrant.
The use of a plastic see-through strip can support students who find this difficult; also drawing the y-axis to the right of the grid can help.

Keywords

coordinate, pairs of coordinates, x-axis, y-axis, origin

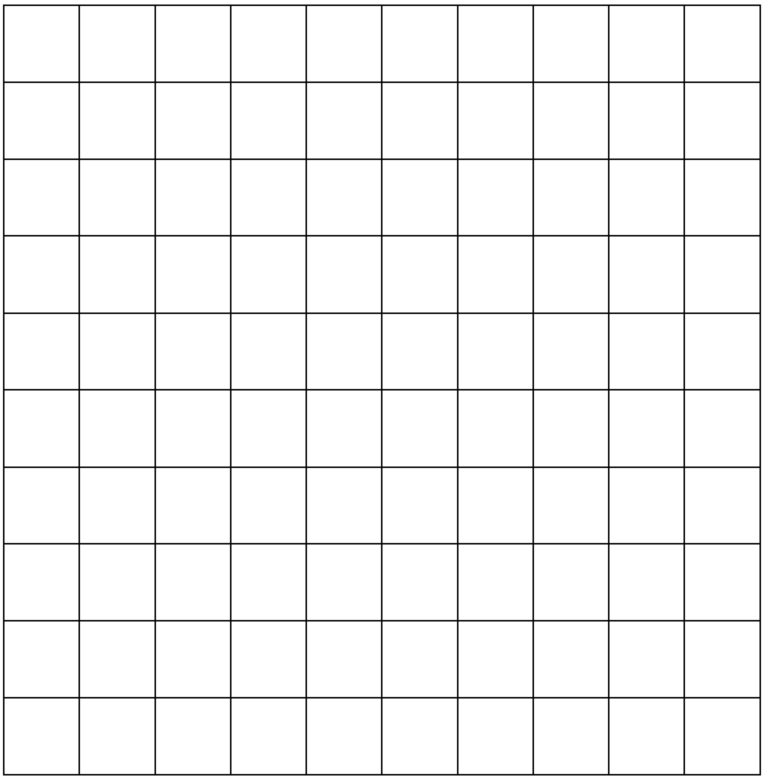
Common misconceptions

Finds it difficult to work with two coordinates: finds one and then drifts from the line while finding the other.
Support as indicated above.

Plots the coordinates in the wrong order.
Using a mnemonic like 'along the corridor and up the stairs' can help.

Coordinates 1 check-up

1. On this grid draw axes for x and y from 0 to 8.
Plot these points: (1,0), (2,6), (7,8).
Join the points to make a triangle.



2. On the same coordinate grid, use a pair of compasses to draw a circle with a centre at (5, 4) and a radius of 4 cm.

3. What are the coordinates of the point where the circle touches the x-axis?

Coordinates 1

Opportunities for reasoning/problem solving – teacher notes

Shape problems on a coordinate grid

Answers

If they need help, you could tell the student that each coordinate pair is used only once.

Squares have vertices at (1, 3), (1, 5), (3, 3), (3, 5) and at (8, 1), (5, 4), (8, 7), (11, 4); the second square is tilted so may be difficult to spot, or may not be identified as a square.

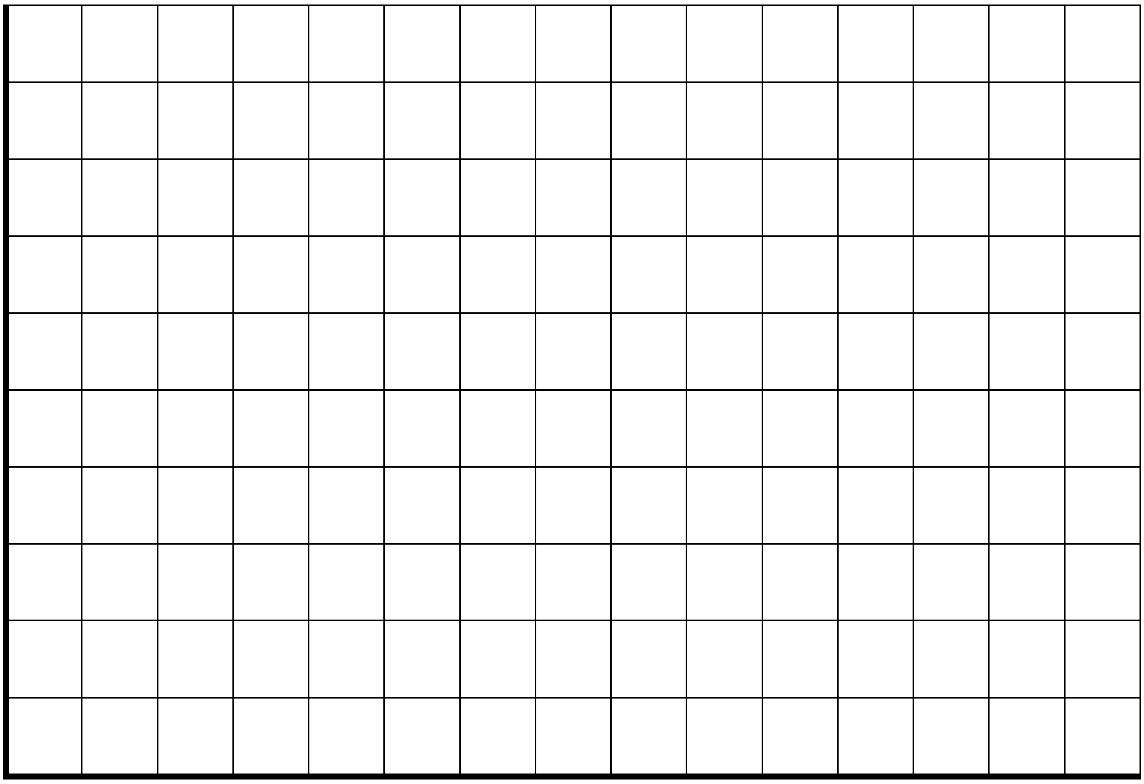
The rectangle has vertices at (12, 1), (14, 1), (12, 7), (14, 7).

The isosceles triangle has vertices at (3, 6), (5, 6), (4, 9).

Each square contains 2 isosceles triangles (or 4 if you count the rotations).

Shape problems on a coordinate grid

Write numbers on the x-axis from 0 to 15 and on the y-axis from 0 to 10.



Mark these points on the grid with small crosses or dots.
It's a good idea to cross each one off as you plot it.

(1, 3), (1, 5), (3, 6), (3, 3), (3, 5), (4, 9), (5, 4), (5, 6)

(8, 1), (8, 7), (11, 4), (12, 1), (12, 7), (14, 1), (14, 7)

Joining some of the points together will make 2 squares.
Which points make one square? (,), (,), (,), (,)
What about the other square? (,), (,), (,), (,)

Can you also find which points join to make a rectangle?
(,), (,), (,), (,)

Another 3 points join to make an isosceles triangle without a right angle.
Can you find them? (,), (,), (,)

There are lots of isosceles triangles with right angles.
How many can you find?

Access Unit 20: Coordinates 2

Objectives

a) Read and plot coordinates in all four quadrants.
The use of a plastic see-through strip can support students who find this difficult; also drawing the y-axis to the right of the grid can help.

Keywords

coordinate, pairs of coordinates, x-axis, y-axis, origin, **positive**, **negative**, **quadrant**

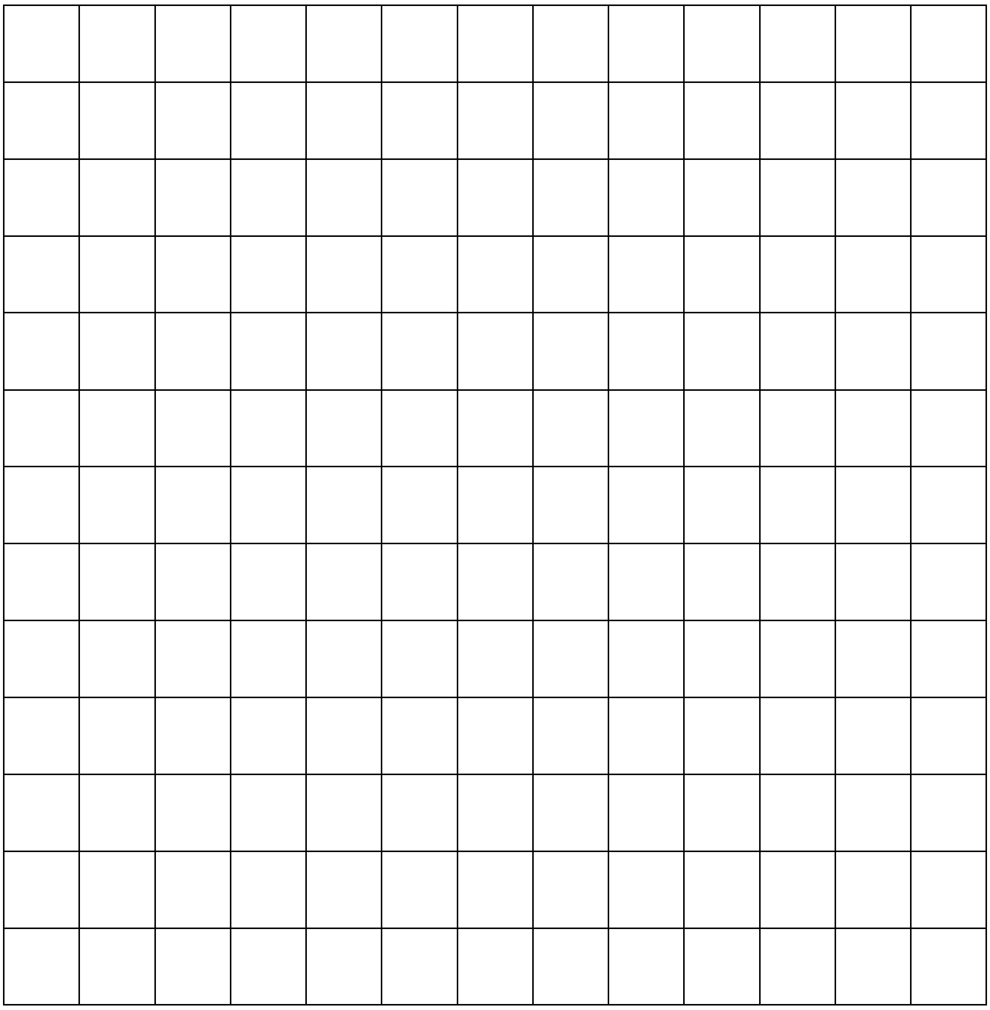
Common misconceptions

Finds it difficult to work with two coordinates: finds one and then drifts from the line while finding the other.
Support as indicated above.

Plots the coordinates in the wrong order.
Using a mnemonic like 'along the corridor and up the stairs' can help.

Coordinates 2 check-up

On this grid, draw a set of x- and y-axes with values from –5 to +5.



Plot these points: (–1, –3), (2, 0), (–4, 0).

These points are at three corners of a square.
What are the coordinates of the fourth corner of the square?

Coordinates 2

Opportunities for reasoning/problem solving – teacher notes

Shape problems in four quadrants

Answers

The corners of the parallelogram are at: (0, 4), (2, 6), (−4, 6), (−6, 4).

The corners of the isosceles trapezium are at: (2, 4), (2, 8), (4, 3), (4, 9).

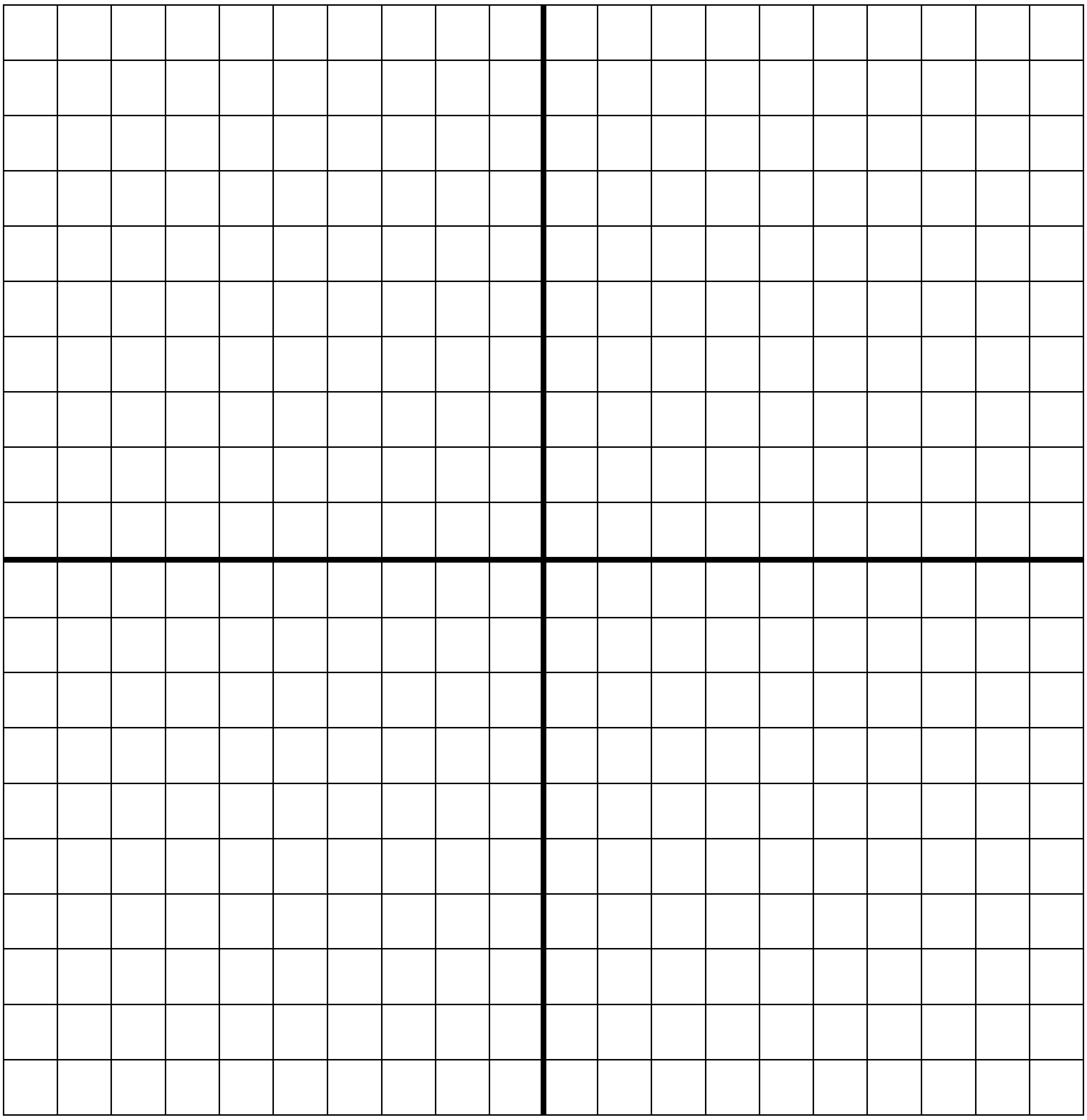
The corners of the rectangle (at an angle) are at: (2, 1), (7, 6), (10, 3), (5, −2).

The corners of the octagon are at: (−3, −3), (−3, −4), (−4, −5), (−5, −5), (−6, −4), (−6, −3), (−5, −2), (−4, −2).

The corners of the hexagon are at: (1, −3), (3, −3), (5, −5), (3, −7), (1, −7), (−1, −5).

Shape problems in four quadrants

Label these x- and y-axes from –10 to +10.



Mark these points with a small cross or a dot. Cross off each one as you plot it.

(0, 4), (1, –3), (1, –7), (2, 1), (2, 4), (2, 6), (2, 8), (3, –3), (3, –7)
(4, 3), (4, 9), (5, –2), (5, –5), (7, 6), (10, 3), (–1, –5), (–3, –3), (–3, –4),
(–4, 6), (–4, –5), (–4, –2), (–5, –2), (–5, –5), (–6, 4), (–6, –4), (–6, –3)

Join some of your points to make:

- A parallelogram
- A rectangle
- A hexagon
- An isosceles trapezium
- An octagon

Access Unit 21: Data handling 1

Objectives

a) Solve a given problem by organising and interpreting numerical data in simple lists, tables and graphs.

Keywords

list, tally, tally chart, graph, table, data, bar chart, block graph, pictogram, survey, questionnaire, label

Common misconceptions

Misreads scale on a graph.

Leaves out zero on a graph.

Data handling 1 check-up

Draw a suitable graph or chart to show this data on favourite food.

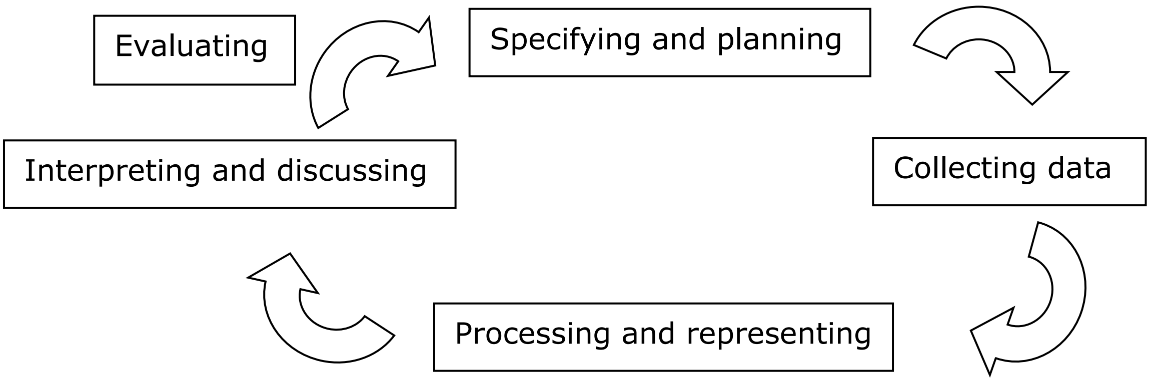
Preferred food	No. of people
Pizza	6
Curry	4
Fish and chips	3
Chicken salad	5
Sausages	7
Burger	6

Data handling 1

Opportunities for reasoning/problem solving – teacher notes

Investigate a chosen problem by collecting data and representing it appropriately 1

Students follow the handling data cycle:



The suggested topic is an illustration of the process, but other topics could be suggested by the students as possibilities.

Investigate a chosen problem by collecting data and representing it appropriately 1

Identify what you are going to investigate

You are going to investigate a problem. This could be about where people went on holiday, and how they travelled there.

Think about the questions you are going to ask

Start by thinking of some questions. You could ask, 'Where did you go on holiday this year/last year?' but it might be better to start with some categories – for example, UK, Europe, further away – as this will make it easier to put your data together later and to represent it on a chart.

What question could you ask about how they travelled? Can you make up some categories for this?

Identify who you are going to ask, and collect the data

When you have got some questions, you need to collect the data (the answers) from a group of people. This might be the other students in your class.

Find a way of representing your data

Look at your data and decide how to show it clearly. You might put it into a table or draw a graph.

Evaluate your work

If you were to do this again, how could you change what you did to make it better?

Access Unit 22: Data handling 2

Objectives

a) **Solve a problem by extracting and interpreting information presented in tables, graphs and charts, including finding averages.**

Keywords

list, tally, tally chart, graph, table, data, bar chart, block graph, pictogram, survey, questionnaire, label, **average**, **mode**, **median**, **mean**, **range**

Common misconceptions

Confuses the different measures of average.

Mnemonics can help: mode = most common, median = middle, mean = means a lot of work. For range, get them to picture a group of people arranged in height order: the range is the difference between the tallest and the smallest, the largest and the smallest value.

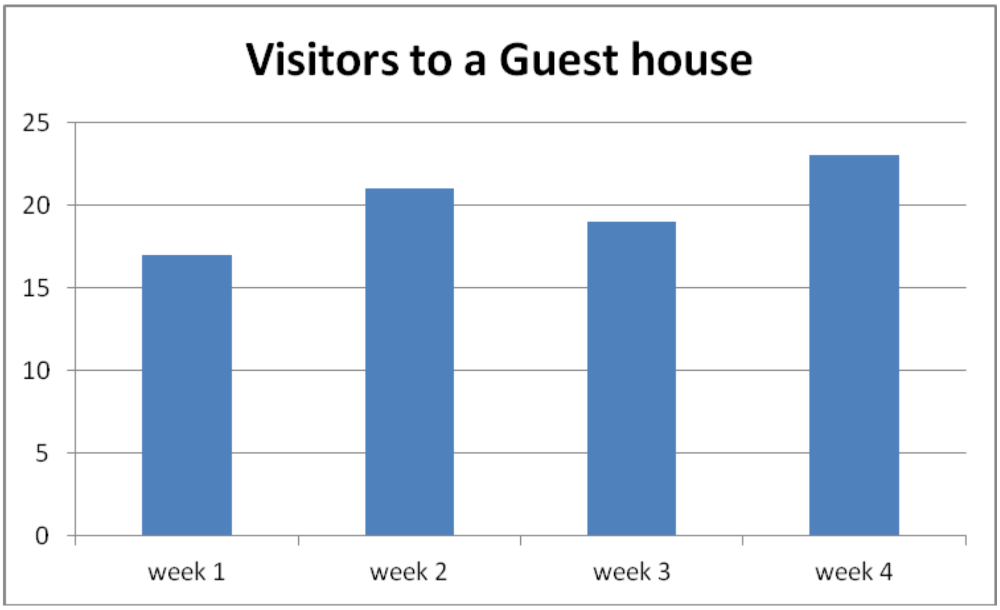
Draws graphs without zero or without consistent scale.

Finds it difficult to decide on scale for own data.

When constructing a questionnaire, asks questions yielding data which is difficult to quantify, e.g. 'Do you watch a lot of TV?'

Data handling 2 check-up

1. This bar graph shows the number of visitors to a guest house over a 4-week period.



What was the mean number of guests during that period?

2. Find the mode of shoe size in your class.

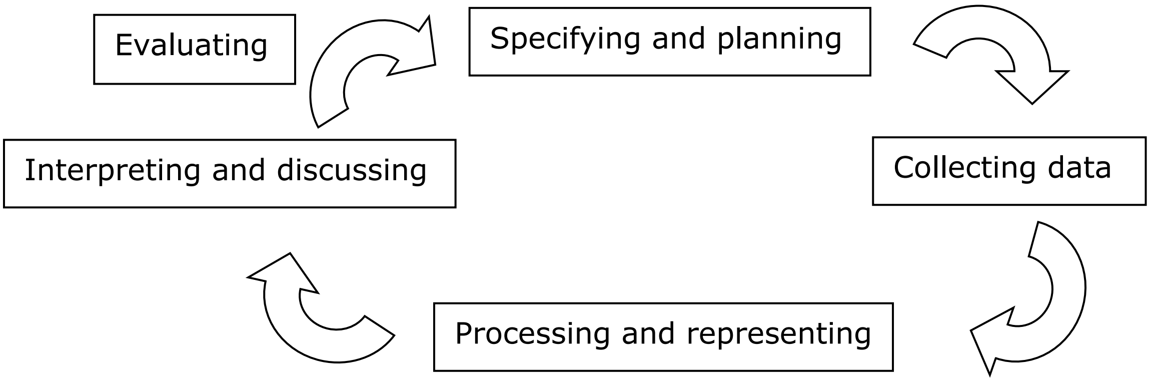
3. Find the median and range of height in your class.

Data handling 2

Opportunities for reasoning/problem solving – teacher notes

Investigate a chosen problem by collecting data and representing it appropriately 2

Students follow the handling data cycle:



Two examples are given in order to illustrate the processes involved, but other topics may be of more interest to the students.

Whatever topic they choose, they should start with a hypothesis which will help to shape the next steps of the investigation.

Investigate a chosen problem by collecting data and representing it appropriately 2

Traffic

You might choose to investigate traffic. Here are some people's thoughts about it (they are called hypotheses):

I think that more lorries than bicycles go along the main road.

I think that 75% of cars have only one person in them.

I think that silver is the most popular colour for cars.

What is your hypothesis?

I think

Recycling

Or you might want to investigate how much people recycle. Here are some hypotheses about that:

I think that about 80% of people recycle paper and card.

I think that fewer people recycle food than glass.

What is your hypothesis?

I think

Collecting the data

When you have decided what you want to investigate (your hypothesis), you need to think about the questions you will ask, and who will take part in your survey.

If you are going to investigate traffic, you will need to decide where you are going to collect your data, when you are going to do it, and what categories you will need on your collection sheet.

If you are going to look at a busy road, you will need to be able to mark things quickly on your collection sheet so that you don't miss any traffic. You will also need to think about where to stand which is safe but gives you a good view.

Next steps

When you have collected your data, you will need to think about the best way to represent it.

When you have created a graph or another representation, go back to what you thought at the beginning and see whether or not your data supports your hypothesis. Do you need to collect more data?

Evaluate your investigation

How could you have improved it?

Access Unit 23: Probability 1

Objectives

a) Understand the language of probability.

Keywords

fair, unfair, likely, unlikely, likelihood, certain, uncertain, probable, possible, impossible, equal chance, improbable

Common misconceptions

Is not familiar with the language of probability.

Cannot distinguish between probable, improbable and equal chance, e.g. thinks that because it might either rain tomorrow or not, there are two possible outcomes, so there is then an equal chance of it raining or not.

Probability 1 check-up

Classify these future events under the headings:
Impossible, improbable, equal chance, probable, certain

It will rain tomorrow.	I will win the lottery this weekend.	I will walk to France in the summer.	The sun will rise on Friday.	If I toss a coin it will come up heads.
If I roll a die it will come up with an odd number.	Next week the day after Monday will be Tuesday.	Next year the first month will be March.	Next year Christmas Day will be in December.	My birthday will never be at the weekend.

Probability 1

Opportunities for reasoning/problem solving – teacher notes

Make up statements which are …
Students sometimes find it difficult to understand the language of probability and this exercise is designed to support their thinking about the different terms.

Follow up questions might be:

- ‘How do you know that’s impossible/certain?’
- ‘How probable is that?’

Make up statements which are …

Make up statements about events in the future which are:
Impossible Probable Certain

Impossible	Probable	Certain

Try to find at least 3 for each heading.
They might be about:

- Winning a prize
- The weather
- What will happen tomorrow/next week/next year
- Tossing a coin or rolling a die
- Where you will go on holiday
- What you might do in the future
- Days of the week or months of the year

Access Unit 24: Probability 2

Objectives

a) **Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts.**
Students will benefit from practical experiments using coins, dice and spinners.

Keywords

fair, unfair, likely, unlikely, likelihood, certain, uncertain, probably, possible, impossible, equal chance, **outcome**, **equally likely outcomes**, **good chance**, **even chance**, **equal chance**, **fifty-fifty chance**

Common misconceptions

Confuses theoretical and experimental probability: thinks that the result of tossing a coin will be exactly equal numbers of heads and tails.

Practical experiments are crucial to show the difference.

Probability 2 check-up

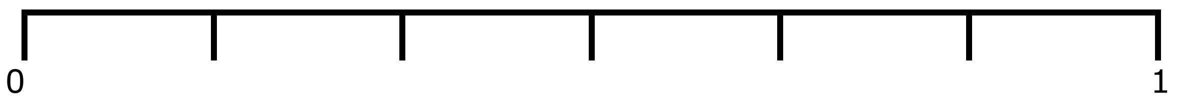
1. How many heads and how many tails am I likely to get if I toss a fair coin 20 times?

Will this happen every time?

What if I toss the coin 100 times?

2. Place the probability of each of these events on the probability scale below.

a. Rolling an even number on a fair dice
b. Rolling a 6
c. Rolling a 7
d. Rolling a number larger than 4
e. Rolling a number >0 and <7



Probability 2

Opportunities for reasoning/problem solving – teacher notes

Dice investigation

Ask the students what they think about the ideas in the speech bubbles before they begin the investigation, and again at the end. If time allows, look at the number of doubles thrown in the experiment to decide whether the third statement is correct.

Practical note – if the students throw the dice into a plastic box, it will be less noisy and the dice are less likely to go on the floor.

The student note sheets give a lot of guidance and support for this investigation.

Likely outcomes are:

- the experimental data follows a similar pattern to the theoretical, but is 'messier'
- students realise very soon that you cannot throw a total of 1 with two dice
- 7 is the most likely total theoretically, but may not be in reality.

Extension - the table of all the possibilities could be used to work out the probabilities of each total as fractions of 36, which can then be compared.

Dice investigation

Here are some people’s thoughts about throwing two dice:

It isn’t possible to get a total of 1 if you throw two dice.

I think 7 is the most likely total when throwing two dice.

It’s not very likely that you will throw a double.

Take two dice of different colours and throw them 20 times.
Record the outcomes in these tables.

Red	Blue	Total

Red	Blue	Total

Record your totals on the chart.

Number of times total occurs

14												
13												
12												
11												
10												
9												
8												
7												
6												
5												
4												
3												
2												
1												
	1	2	3	4	5	6	7	8	9	10	11	12

Total of two dice

Can you see any patterns in your results?

Throw the two dice another 20 times and record the totals below.
Add them to your chart above.

Has throwing the dice more often changed your results? How?

Now look at *all* the possible totals. Fill in the table with the totals you would get with different dice scores. A few have been started for you.

Red dice score

Blue dice score

+	1	2	3	4	5	6
1						
2			5			
3					8	
4						
5	6					
6						

Now put these totals on a chart like the one you did before.

Number of times total occurs

14												
13												
12												
11												
10												
9												
8												
7												
6												
5												
4												
3												
2												
1												
	1	2	3	4	5	6	7	8	9	10	11	12

Total of two dice

What is the same about the two charts? What is different?